ARE SCORES MAPS? A CARTOGRAPHIC RESPONSE TO GOODMAN

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ABSTRACT

Nelson Goodman's theory of notation attempts to provide an ambitious, unified account of how systems of symbolic representation preserve and transmit information and how they differ from pictorial depiction. However, Goodman's account of music and dance notation has proven unpopular, with some critics objecting to the rigor with which scores and musical symbols are assumed to designate musical works and their constituent elements. This paper reconsiders a Goodmanian account of a music notation system in the light of recent philosophical work on maps and map-like cognition. Specifically, I propose that scores do not act as compound symbols that uniquely designate musical works. Instead notational components of scores are better understood as contingent surface-level features leveraged by an underlying map-like representational structure. On this account, scores are seen to be highly conventionalized maps, and the notational symbols of scores constitute just one of multiple modes of representation and depiction harnessed by this framework. Finally, I consider several contemporary examples of music notation and discuss how a cartographic theory of notation may provide novel insights into the graphic design considerations of these scores, particularly those that rely on new notation platforms such as graphic design software or animation, where depictive and symbolic strategies are frequently hybridized.

1. INTRODUCTION

Music in the European tradition has frequently been compared with language, and insofar as the score and notation are assumed to be the primary communicative vehicle of a work of music or dance, scores have also been compared to the written word. Nelson Goodman's theory of notation represents a highly refined version of this argument; music notations are analyzed as having the same form and function as the symbolic representations of languages. In this paper I propose that a cartographic system of representation is a plausible alternative to a sentential theory of music scores. I follow Elisabeth Camp, who has argued on both philosophical and neurological grounds that a map-like form of cognition is an alternative model to the "Language of Thought" argument, which holds that thought must be language-like [1]. In what follows, I will first summarize Goodman's theory of notational scores and objections to the theory. I will then provide an outline for a cartographic theory of scores (although space does not permit a complete exposition of the argument), and I will conclude with several practical examples of score design problems that might benefit from an analysis of the logic of and graphic design in maps.

1.1 Languages of Art

Nelson Goodman's theory of music notation arises from his broader interest in symbols, which is mainly set forth in his 1968 book, *Languages of Art*. Subtitled "An Approach to a General Theory of Symbols," this ambitious project sought to establish a unified analytic theory of symbols that would be broad enough to encompass the many disciplines in which they function, including natural languages, visual arts, music, dance, and the sciences.

Although appealing in its scope and explanatory power, Goodman's project has been unpopular with philosophers as a theory of music or dance notation and has been largely dismissed by music theorists and composers as well. In part, this resistance stems from the rigidity by which Goodman believed scores identify compositions. According to the theory, only strictly notational [1] elements of a score are preserved with accuracy over successive reproductions of a score, and only performances that comply fully and exactly with the notational parts of a score can count as valid performances of a work. Experimental or graphic notations, which do not rely primarily on notation "scheme," cannot be trusted as preserving a work in a strict sense

Goodman's theory in fact sets such a high bar to work identification (the presumed purpose of scores) that on Goodman's account we likely never hear a genuine performance of any musical work or score, a fact not lost on many of his critics. In separate papers Paul Ziff and William Webster have convincingly argued that Goodman's theory of music notation failed to reflect the meaning and practical usage of scores, with Ziff additionally suggesting that Goodman overlooked the degree to which scores can only be accurately interpreted within the context of a particular performance-practice tradition [2, 3]. James Elkins has questioned whether the marks (the specific manifestations of notation on the printed page, as apposed to the interchangeable symbols) of music notation are truly indifferent, that is, whether the shape of the score elements, apart

[1] Goodman's use of the term "notation" refers to a strict usage of the term that differs substantially from the vernacular meaning. This distinction will be elaborated in section 1.2.

from their symbolic meaning, might have a significant effect on how musicians interpret a score [4]. Composer Jean-Charles Franois takes issue with Goodman's assumption that scores identify works (at least in the modern era) at all, preferring to consider the realization of a work alone to constitute that work [5]. Virginia Anderson notes that Goodman's rejection of graphic scores as non-notational leaves them in a kind of limbo, being far too score-like in their usage to be considered improvised compositions, while also apparently serving no work-preserving function, according to Goodman [6].

Despite its seeming shortcomings and paradoxes, Goodman's theory of music notation deserves reevaluation. Goodman brings attention to several often-overlooked questions: what kinds of information can be preserved in scores with fidelity? Is some score information more critical to the essence of a musical work than other information? And can studying notation give us insights into what musical parameters composers mean to preserve and which, if any, are contingent parameters? Additionally, Goodman's theory of notation allows music notation to be evaluated in the context of a general practice of notation in all disciplines, including, for example, scientific notation and data visualization. As composers increasingly make use of new tools for notation, including vector-based graphic design software and computer animation, it is important to develop philosophical paradigms for analyzing these works in a multidisciplinary graphical context.

While a purely notational account of contemporary scores may be implausible for reasons that will be elaborated in section 1.3, recent philosophical work on map semantics suggests that Goodman may have been right in his account of notations but mistaken about the fundamental representational modality of scores. In a discussion of maps, Goodman observed that road maps rely on a mix of analog and digital symbology. In a similar vein, John Kulvicki has observed that maps are "picture-language hybrids." It is striking that Goodman did not explicitly draw a parallel between the hybrid representation strategy of maps and that of scores. In emphasizing the notationality of scores, Goodman downplays the importance of other modalities of representation in scores, claiming that music notation "comes as near to meeting the theoretical requirements for notationality as might reasonably be expected of any traditional system in constant actual use, and that the excisions and revisions needed to correct any infractions are rather plain and local." This is not plausible, especially in the case of most contemporary scores where pictorial representations often significantly supplement or even replace traditional notation symbols.

In reframing music notation as a contingent feature of scores, a certain rigid conception of score-preservation and work-preservation must be sacrificed. However, if scores are in fact highly conventionalized maps, this account gains the ability to explain many special cases of contemporary score-making, use, and interpretation while revealing ways in which the syntax and semantics of maps function to represent a musical work through a sophisticated multimodal scheme.

1.2 Notation

For Goodman, notational systems are systems of symbols that represent things with a particular kind of fidelity. Notations section off and label certain parts of the universe, allowing information to be preserved without loss of accuracy due to subjective evaluation or imperfect reproduction. This distinction is the difference, for example, between recording a particular geometric angle in degrees or radians versus recording that same angle as a line drawing of an angle. Given consistent measuring equipment, an angle notated in radians can be reproduced with absolute fidelity, whereas an image may be degraded by subsequent reproductions.

An example of a strictly notational system is chess notation. At least one type of contemporary chess notation completely eliminates ambiguity from the recounting of a chess match. According to the "Figurine Algebraic Notation" (FAN) system of chess notation, each square on the board has a unique and discrete Cartesian coordinate. Furthermore, a unique pictogram represents each piece on the board (with the exception of pawns which are described by their rank and capture history). For example, moving the white queen two squares forward from her starting position is indicated in FAN by her symbol and destination coordinates, ♕d3.

Although we commonly refer to many kinds of symbolic depictions as notations, Goodman restricts this term to symbolic systems that fulfill strict criteria. The importance of defining a technical sense for the word notation, which may depart from the vernacular use of the word, is to explicate how and in what cases we can be confident that a symbol refers without ambiguity.

In order to be notational, the symbols that comprise a notational scheme must fulfill five criteria. Goodman's first two criteria relate to the syntax (or representational form) of symbols, while the remaining three criteria relate to the semantics of the symbols (or the content of these expressions). As Camp points out, this distinction between form and content is normally associated with linguistic expressions, but in the present case, it applies to any notational scheme and, as we will see later, is also relevant to the representational modality of maps. Goodman's five criteria for notational systems are as follows:

1. **The constituent symbols of a notational system must be disjoint (or "character indifferent").** In other words, marks that stand for equivalent symbols in a notational system must be capable of being exchanged without syntactic consequence. For example, in Figurine Algebraic chess notion, no symbol ever counts as an instance of more than one symbol in the system; e.g., there is no mark that stands in for both the symbol for the white queen and the symbol for the black queen. What matters is not that two characters be easily differentiated in practice—symbols may still be disjoint even if they are difficult to distinguish; such a notation would simply be an inconvenient notation, not an invalid one—rather it is the quality of belonging to only one class of

marks, (containing instances of a single symbol) that makes a notation disjoint or not.

2. **Symbols must be finitely differentiable, and such symbols are said to be "articulate."** That is, it must be theoretically possible to ascertain whether any two symbols in the scheme are disjoint. Goodman uses the example of a notation system composed of straight lines where lines are different symbols if and only if they differ in length to any degree. Since no test can ever guarantee that two lines do not differ in length by an unascertainably small margin, it can never be determined whether the lines are disjoint. Hence such a system is not articulate.

3. **The extension (or compliance-class) of a symbol must be unambiguous.** That is, the semantic referent of a symbol must be uniquely picked out by that symbol. In other words, regardless of when or in what context a symbol is used, the object represented by that symbol will always be consistent. For example, in Figurine Algebraic chess notation, the white queen is always referred to by the symbol ♕regardless of time or context.

4. **The semantics of the notational symbols must also be disjoint.** The set of objects to which a symbol refers may not overlap with the set of objects referred to by another symbol, e.g., redundancy within the field of reference is not permissible within a notation system.

5. **The compliants of a symbol within a notational system must be semantically finitely differentiated.** That is, it must be theoretically possible to determine that an object fails to comply with any given symbol in the notation.

Syntactically, symbols within a notation system may be composed of an indivisible unit ("atomic symbols") or composed of multiple atomic symbols ("compound symbols"). On Goodman's account, a musical score as a whole is a compound symbol that uniquely identifies a particular musical work. The purpose of a score is therefore to identify a particular musical performance with the musical composition of which it is an instance.

> "A score, whether or not ever used as a guide for a performance, has as a primary function the authoritative identification of a work from performance to performance" [6].

In order to uniquely identify a performance as an instance of a work, the score, as a notational symbol, must conform to Goodman's five criteria, which further entails that at least some relevant portion of the score must itself be based on notational symbols. For example, Goodman identifies pitch and rhythmic notation (the latter only in practice rather than in terms of its theoretical syntax) as being notational, at least as far as can be expected for a notational system in "traditional, actual use." On the other hand tempo indications (and presumably dynamics, glissandos,

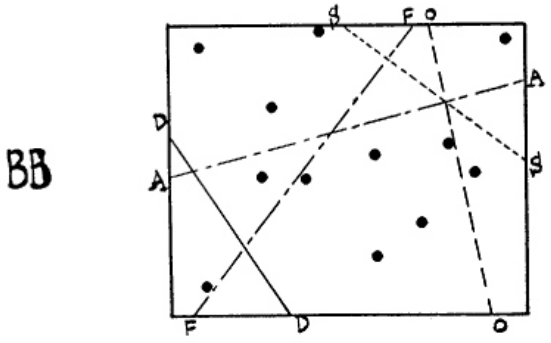

Figure 1. Without "stipulation of minimal significant units of angle and distance," p. 53 from John Cage's *Concert for Piano and Orchestra* from 1960 is not syntactically differentiated on Goodman's account. [2]

and much else), being syntactically dense, cannot be used to uniquely identify a score. Work identification cannot hang on any of these properties therefore, neither can any graphic score (Figure 1 in Goodman's own example) count as a score, since it contains no notational information.

1.3 Critical Response to Goodman

Although Goodman's economy of means is elegant, his conception of a score requires that, strictly speaking, we must reject the authenticity of any performance of a musical work that fails to conform to the minutiae of the relevantly notational elements of the notation. An imperfect performance of a work is not, in a strict sense, a performance of that work, because the score only represents performances that fall within its compliance class. Although we are free to speak casually of a performance being a performance of such and such a work, in a strict sense Goodman is adamant that a performance of a composition is only a realization of that composition if it is an exact realization of the score's notation.

> Since complete compliance with the score is the only requirement for a genuine instance of a work, the most miserable performance without actual mistakes does count as such an instance, while the most brilliant performance with a single wrong note does not. [...] If we allow the least deviation, all assurance of work-preservation and score preservation is lost; for by a series of one-note modifications, we can go all the way from Beethoven's Fifth Symphony to Three Blind Mice [6].

The strict sense in which a score identifies a work according to the score-as-symbol theory leads to some counterintuitive results. For example, since tempo marks are syntactically dense and hence not one of the relevant notational elements of a score on which work identification hinges, a

[2] Page 53 from John Cage's *Concert for Piano and Orchestra* is reproduced by permission of Edition Peters.

performance may still be an instance of a work even if it is played vastly faster or slower than the composer intended. The score for Beethoven's Ninth Symphony would theoretically still identify a performance of that work as an instance of the work even if the Ode to Joy were played over the course of an entire week or as a blur of nearly unrecognizable noise lasting only seconds, so long as the performers didn't actually miss or change the notes and rhythms of the work relative to each other.

It is also unclear exactly what it would mean for a musical work to be played according to the notational elements of the score. Although pitch and rhythm (in practice) are notational (at least in common-practice period notation) both of these parameters vary considerably depending on the performer, the circumstances of the performance, and the musical context in which the relevant passage occurs. For example, pitches in piano scores designate 12-TET tempered pitches, some of whose intervals are "out of tune" when compared to Pythagorean intervals. String players generally tune to Pythagorean intervals, except when they are playing with a keyboard instrument. Perhaps pitch notation is only notational in the context of a specific ensemble or for a specific player, but this too is challenged by the ubiquity of small pitch variations within even a short passage of music; in tonal music there is a tendency to raise the "leading tone" slightly; diminished tones are often played flat. Analyses of phonograph recordings of violin music found that violinists deviate from tempered pitches by 0.05 tones about 60% of the time and by 0.1 tones about 32% of the time [7].

The problem is not that notation requires absolute precision; semantic finite differentiation is sufficient to allow some tones to be identified as complying with no pitch in the notation (or at least this was the case before the ubiquity of microtonal music). Rather the problem for a notational conception of scores is that the symbolic representation of pitch seems to mean different things at different times, certainly between different instruments, but also even within a single phrase of music.

Imprecision in performed rhythms is pervasive and well-documented as well. Gabrielsson reports deviations of between 10-20% from the notated rhythm within two phrases of a Mozart piano sonata [8]. Various hypotheses are proposed for this variation ranging from expressivity to perceptual compensation or motorcontrol factors [9], but certainly such large rhythmic deviations bring the semantic disjointness of the notation into question.

Goodman gives us few hints as to how these problems might be reconciled. His project is fundamentally premised on providing a strict definition of score compliance, the criteria by which a performance may be judged to have been a performance of a specific musical work. Insofar as it is merely *impractical* to comply with all the notational information conveyed by a score, this is not a challenge to the theory. There is value, perhaps, in demonstrating the futility of ever actually performing a work of music according to a notation. (Some authors have in fact taken Goodman to have demonstrated that every musical performance is a kind of improvisation in a sense [10].)

Beyond various problems with the Godmanian notationality of music notation, the account doesn't seem to capture something important about the way musicians and composers interpret scores. A conception of score interpretation premised on producing precisely the correct referent of every notational symbol in a score seems stiff and contrived, what musicians refer to as "playing the notes" as apposed to performing music. Score interpretation has much more to do with context, finding how all the parts fit together or following a musical line or phrase. A change in tempo influences not only how we interpret the temporal symbols in the marked passage but also how we think about other passages of music, the purpose of that part of the music in relation to others, perhaps even how we think about the representational strategy of the score as a whole. Each symbol in a score affects the symbols around it and the work as a whole. This codependency of spatially and temporally representative abstract parts is a key feature of maps which will be discussed further in section 2.

2. A CARTOGRAPHIC THEORY OF SCORES

In arguing that scores are maps, I wish to make a claim about their syntactic and semantic strategy of representation rather than about the historical purpose or usage of maps. A "map" is therefore a broadly construed class of representations that may overlap in certain cases with what we might be more inclined to call graphs, infographics, diagrams, schematics, and charts. Camp argues that maps fall somewhere between pictorial and sentential modes of representation, and with some important qualifications, scores strike a similar balance.

Scores are maps that are isomorphic with the spatial and temporal structures of the musical works they represent, while other graphical features may be purely contingent or incidental. This highlights an interesting property of maps: they need only be isomorphic with regard to a subset of the properties of the space they represent. A true subway map must be isomorphic to the order and correlation of subway stations, but almost every other property of the landscape can be omitted or abbreviated symbolically. What aspects of the world are represented and what aspects are omitted or stylized would seem to have a great deal to do with the power of maps to expand and clarify our understanding of specific spatial relationships. Camp highlights that the choice of features depicted is connected to the practical function of a map.

> "[...] typically this spatial isomorphism itself only captures functionally salient features of the represented domain: for a road map, say, only streets and buildings and not trees and benches.[1]

Unlike road maps which represent a certain geography, albeit from a "disengaged, 'God's eye' perspective" [1], scores represent an array of highly structured acoustic morphologies and performative actions through two-dimensional, visual conventions. In essence, scores translate a specific subset of acoustic and temporal features of their referents

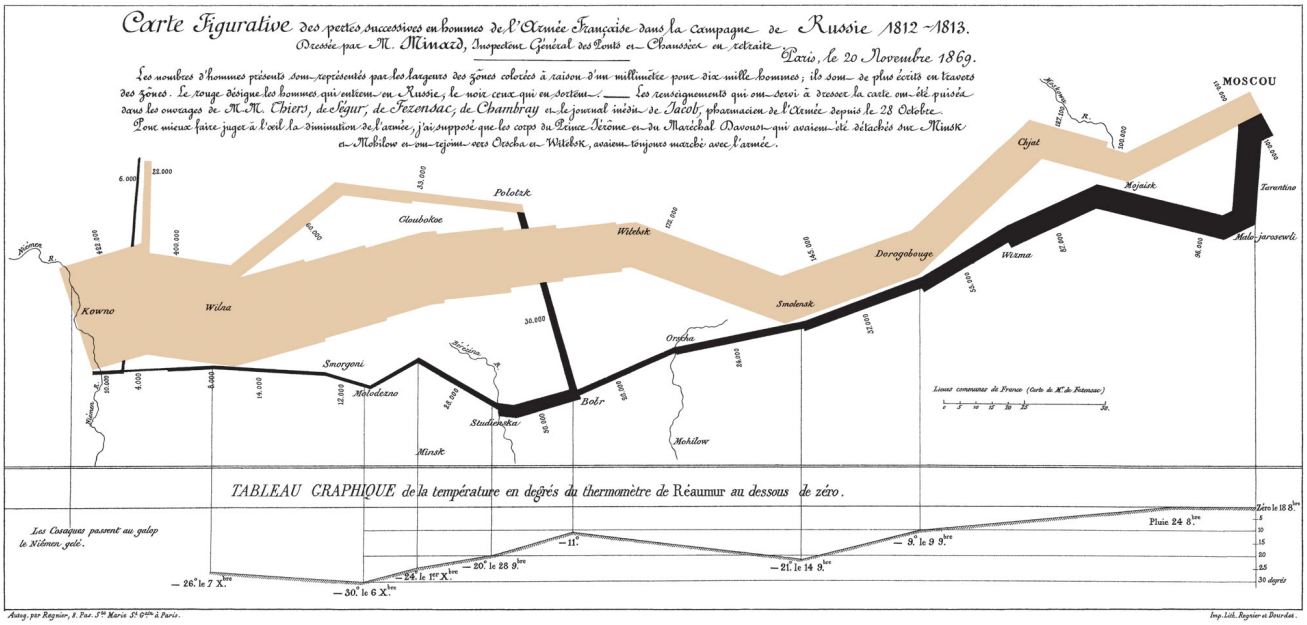

Figure 2. Charles Minard's 1869 graphic representing Napoleon's disastrous Russian campaign and an excellent example of a cartographic representation of spatial and temporal events (months, shown with Roman numerals, are correlated to temperature and spaced according to the distance between landmarks).

to a visual representation. Because of this, what constitutes isomorphism is far more conventionalized in scores than in most maps, depending on cognitive metaphors to translate back and forth between spatial and temporal domains rather than simple visual similarity. Within these conventions, isomorphism is preserved however. "Higher" pitches appear visually higher on musical staves; rhythms are ordered as they occur in time from left to right; in percussion music, instruments are grouped as they appear before the performer, with each instrument in a collection assigned to a line on a special staff (or a syntactically disjoint symbol, e.g. a notehead of a certain shape).

As in many maps, scores can use sentential representation, and through the map-like structure of the score, these expressions gain the ability to refer to specific temporally and spatially locatable features. Performance directions can be far more conceptual than can be easily expressed either through pictures or diagrams, with Pierre Boulez, for example, calling on the performer of his Second Sonata to play in an "exasperated" or "strident" manner or, later, to "pulverize the sound." Through the map-like scheme, these abstract invocations are applied only to certain sections of a work, thereby increasing the expressive power of language beyond the contents of the sentential expression.

Scores are not themselves sentential in structure however, since they lack the extremely hierarchical and abstract structure of language. Disregarding aesthetic or stylistic concerns, the discrete parts of a score can generally be rearranged with a great deal of freedom, and musicians even refer to these parts in spatial terms. A musical line can be "inverted"; melodic lines are said to be "close" or to "cross" or contain too many "leaps"; harmonies are said to "revolve" around a "harmonic center."

2.1 Representing Objects and Events

The most striking characteristic of scores—that they represent objects and events with regard to time—is an uncommon but not wholly neglected, feature of maps. Gail Langran and Irina Vasiliev have documented cartographic practices of depicting time, with Vasiliev dating the earliest examples back to at least the 18th century [11, 12]. We are most familiar with temporal map-like depiction from animated maps such as weather maps, traffic maps, or animated subway maps (although it is debatable in what circumstances animated maps represent time or whether they actually *depict* time through a real-time change in the image).[3] Among printed maps, excepting scores, representations of temporal processes are generally only achieved crudely. Maps of historical battles often depict the movements of military units with arrows. Maps of population growth and migration show the expansion of species or living organisms over geography in a very general way.

A notable early exception to this is Charles Joseph Minard's illustration of Napoleon's 1812 invasion of Russia (Figure 2). Unusually for a map, this illustration correlates time with multiple other domains of information, showing landmark dates during the disastrous fall and winter retreat correlated to both temperature and geographical movement of the army. Something similar usually occurs in scores, where musical time flows differently according to circumstances, being modified by tempo indications or rubato, for example, while at the same time, temporal events are tightly bound to a vast array of spatial and performative information.

In a brief survey of the philosophical literature on the spatio-temporal analogy, Robert Casati and Achille Varzi

[3] Further research is needed to establish a theory of animated maps that might inform the design of animated scores.

note two schools of thought [13]. Bertrand Russell [14], Alfred Whitehead [15], and Willard Quine [16] generally held that physical objects and temporal events are highly analogous, while the disanalogies stance is tokened by David Wiggins [17] who objected that the boundaries of spatial objects may be explored while this is not obviously true of temporal objects. If true maps are admitted as a means by which a continuant is explored, then temporal maps, and particularly scores, offer an interesting challenge to Wiggins' contention. Perhaps scores do not offer insight into specific events, but they do allow inferences about planned or hypothetical events such as the performance of a particular piece of music. Casati and Varzi outline a "formal map," which is to informal maps as formal logical languages are to vernacular language. A temporal referent would not appear to be any barrier to the creation of a "formal score" with an analogous form.

As with a cross that represents a church on a road map, expressive or technical directions in scores are represented by arbitrary designators (symbols or words) that stand in complex relationship to numerous other features of a musical work. These designators fall before or after other features; they apply to specific instrumental parts; and they last for finite durations, dividing up the temporal space of a work as a map is divided between "land" and "water." As with symbols on a road map, we may even be unaware of the meaning of a symbol, but as long as we are familiar with the isomorphic, spatial strategy employed by the map, we can make valid statements about that symbol in relation to others in its vicinity.

Interestingly, Gennady Andrienko, et al., note a temporal corollary for "Tobler's first law of geography" ("everything is related to everything else, but near things are more related than distant things" [18]). Referred to in its spatial manifestation as "autocorrelation," the principle that closely spaced spatial features are dependant on one another is seen in the temporal domain as well in the connection between past, present, and future. Temporal features run forwards and backwards through time, with experience of the past and anticipation of the future both informing the present [19]. This principle certainly holds for scores, where for example "courtesy accidentals" are used to confirm the cancellation of a change in pitch that occurred earlier in a passage of music.

2.2 Conventions of Representation in Maps

Goodman denied that pictures depicted through resemblance with their subject. Rather, Goodman believed that depiction was almost entirely a matter of artistic convention. Without wishing to take a position on this question here, the same cannot hold true for maps. Maps may be conventionalized to a very high degree in their non-semantically relevant properties. A 2012 New York City subway map redesigned by Max Roberts uses only sections of concentric circles, abstracting away nearly all information about absolute distance or geographic movement vector [20]. The interaction between the syntactically relevant representative components of a map must still stand in an isomorphic relationship with the depicted properties of the landscape however, or else the map is inaccurate. In the case of a subway map, the subway stations must occur in the correct order, although a wide variety of symbols and labels may stand in for the stations and the subway lines themselves. Concerning the symbolic constituents of a map, Camp has observed that they too exhibit some limits to their abstraction:

> "[A]lthough maps employ discrete syntactic constituents with a significantly conventionalized semantics, there's still a significant interaction between their formal properties and mode of combination and what they represent. Nonetheless, the only strong constraint on the icons employed by cartographic systems, and on their potential semantic values, is that the icons' own physical features can't conflict with the principle of spatial isomorphism. Thus, one can't represent a street with a circle, not because it would be too arbitrary, but because this would make it impossible to place the icon in a spatial configuration that reflects the spatial structure of the represented content: for instance, one couldn't depict two streets as parallel, or as intersecting" [1].

Maps differ from pictures in that they abstract away much of the detail of pictures, increasing comprehension by replacing complex depiction with symbolic representations while preserving certain relevant spatial relationships between these constituents. Cities and towns are replaced with pictograms or labels. Roads and highways are lines of different colors. Colored patches represent areas of water or forest.

Different types of maps abstract different features and range in their level of detail from, as Camp points out, Google Maps renderings that allow for satellite and street-view images to be overlaid over roadways (at the less abstract end of the spectrum) to subway maps and seating charts (some of the most abstract maps in common usage). By depicting certain properties as highly isomorphic while others details are omitted or stylized, map designers affirm the importance of certain kinds of information and relationships while downplaying other details. It is vital to the success of a map that it be isomorphic in the properties most vital to a map's intended usage. A nautical map must reflect the depths of oceans and waterways, while a road map need only show the location of water. Depicting the depth of water on a road map would only serve to distract from the map's intended purpose.

Similarly, composers adopt a position on what criteria are vital to the essence of a musical work when they prioritize certain types of representation in their scores. These choices, which I refer to as "work-preserving criteria" and "score-preserving criteria," suggest a different model of work preservation from that advocated by Goodman. Instead of basing work preservation on a score functioning as a compound notation for a work, works preserve only relevant isomorphic features, and these are used to navigate a temporal and acoustic space suggested by the composer.

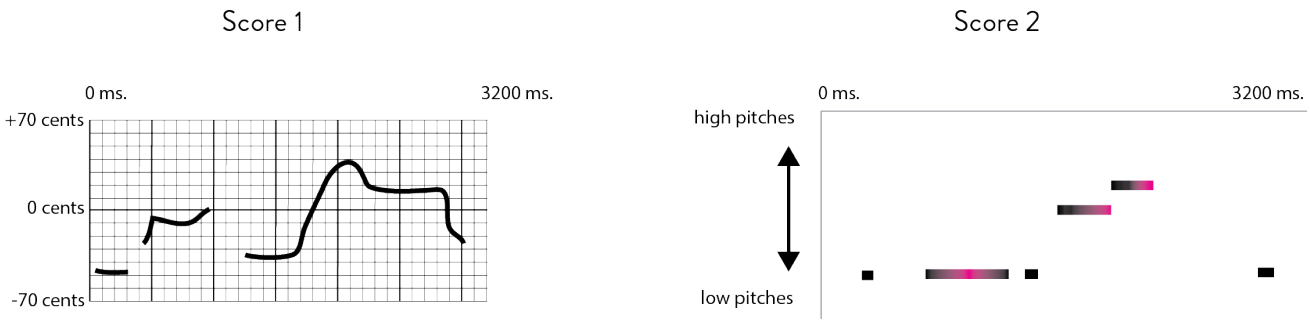

Figure 3. Two hypothetical scores committed to two different scales and exhibiting very different score and work-preserving criteria.

3. SCORE DESIGN CONSIDERATIONS

Although the philosophical underpinnings of a cartographic theory of scores require a more thorough exposition elsewhere, my intention here is to consider a practical theory of cartography that can yield insights into notational practices. In what follows I will discuss two aspects of map idioms: scale and coverage.

3.1 Scale and Preservation

Regarding score preservation in graphic systems such as Figure 1, Goodman worries that "however small the inaccuracy of reproduction, a chain of successive reproductions can result in departing to any degree from the original." Finitely differentiated symbols do preserve information better across successive reproductions; while as the symbols that comprise the notation are identifiable, their compliance class is fully intact.

Slight imperfections of reproduction have less semantic significance on a score-as-map conception. The accuracy of a map is only valued according to how it is used. A key to informational density, the scale of a map is intended to give some indication of what kinds of uses a map might be good for. If we wish to know the travel time required to drive from Bremen to Stuttgart, we may be happy with a map that represents distance in kilometers. On the other hand, if we must know the location of the gas line entering a house, only a map or diagram representing distances in inches or centimeters will suffice. The degree of inaccuracy we are willing to accept in a map depends on what we want to do with the map. Similarly, different musical works accept different levels of inaccuracy, and according to the conventions of the style, we may or may not be inclined to accept a particular performance as a genuine instance of a work depending on the degree to which the performance departs from the score.

The representational scheme chosen by the composer always necessarily prioritizes certain kinds of accuracy of reproduction while deemphasizing other less salient syntactic components. Furthermore the choice of a particular scheme implies that certain syntactic components will receive more consideration in making judgments of work preservation than others.

For example, Figure 3 represents two hypothetical scores. "Score 1" leverages Cartesian graph notation and allows for nuances at least down to tens of cents. By choosing to represent this kind of detail, the composer implicitly takes a position on the "scale factor" for the score, which in turn has implications for the score-preservation criteria for the work. The score is not fully notational by Goodman's standards. For that we would need a syntax for the contour line including notation for angles, path lengths, etc. However, we can infer that an existential threat to the score would be one that prevents us from interpreting the contour paths with accuracy on the order of tens of cents. We can also make map-like intuitions that will constrain the inaccuracy of the contour paths within the limits of the scale factor. For example, we can note that the first contour in the work (beginning between 0 and 100 milliseconds) is in the third space up from the bottom of the graph and is just touching the third line up from the bottom.

It is true that over successive reproductions of the score, the exact path traced by the contour line may be affected by successive inaccuracies in the reproduction process (as in Goodman's score-preservation challenge to Cage, Figure 1). However, by not defining the contour line's path more strictly, we should understand that the composer is implicitly assenting to the proposition that score preservation still holds *so long as the contour line does not depart too far from the constraints of the scale.* In other words, a change of 20 cents in contour line morphology would destroy score preservation. A change of 3 cents (hardly visible on the score) does not threaten score preservation, and any change much more than this will be rapidly detected in relation to the graph. The graph lines, like cartographic symbols for longitude and latitude, are notational and therefore limit the degree to which the analog parts of the score could conceivably deviate from the manuscript.

In Figure 3, "Score 2," the composer has implied a different scale and hence very different score-preservation criteria. Here our only indication of pitch is a range between high and low. By constraining pitch only loosely, the composer implicitly assents to the proposition that precise pitch is not a factor that affects score preservation. Rather, the map-like syntax implies that the ordering of pitches is mandatory, and changing the order of high and low pitches would pose an existential threat to score preservation. Similarly, by not providing a graph by which to compare note lengths in milliseconds, the composer is assenting to a scale factor that requires performers to follow

Figure 4. Detail of freeway map of Los Angeles redesigned by Peter Dunn. [4]

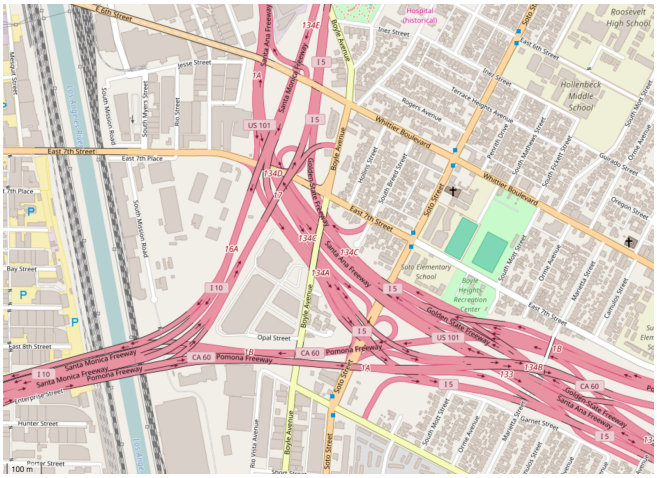

Figure 5. A typical color scheme for road maps is based on a highly isomorphic representation of locatable features but may be disorienting for wayfinding [5]

only very approximate note durations. In this case, playing a notated short note for a longer duration than a notated long note would pose an existential threat to score preservation, but minor inconsistencies in note duration are tacitly permitted.

3.2 Coverage and Degrees of Freedom

The notation in Figure 3, "Score 2," differs from "Score 1" in another important regard: the use of a third color (in fact a color gradient) allows this notation to refer to an additional "degree of freedom," perhaps dynamics or a timbrel effect.

Kulvicki refers to degrees of freedom as features of a map that, once introduced, have communicative significance across the relevant portion of the map [21]. In Kulvicki's example, for instance, a simple map may be silent as to whether a green "land" area of a map is flat or mountainous, but once a squiggly line is introduced to represent hilly terrain, then unblemished green has an additional meaning within the degree of freedom that encompasses the binary "hilly" versus "relatively flat" terrain. Each degree of freedom a map represents commits that map to representing the null value for that degree of freedom wherever a space is left unmarked; if a map commits to representing towns symbolically, the absence of a "town" symbol commits the map to an absence of a town at that location.

Like maps, scores are agnostic with regard to all degrees of freedom save the ones introduced into the score by the composer. Scores that represent only pitch information say nothing at all about rhythms, or rather they imply that durational information must be improvised up by the performer, either by relying on conventions or through other, perhaps sentential, instructions. (This is the case for some of John Cage's freer "Number Pieces," such as *Four*[3], for example.) Once a notation for rhythm is introduced into a score, however, passages without rhythmic notation imply a special significance; The scale of a score, the level of detail it commits to representing (i.e., its score-preservation criteria) are degrees of freedom, because they represent the detail that a composer has represented as important for the particular work of which the score is a map. In contemporary notation where performance practice fills in very little for a performer, a score that is agnostic as to note durations implies that the composer is explicitly declaring durational plasticity to be a score-preserving feature of the work. To represent specificity with regard to some features in conjunction with agnosticism about others is to make a statement about what features of a work are valuable to the composer of that work.

Kulvicki also defines an "incompatibility constraint": for well-formed maps "incompatible locatable features represent incompatible qualities." For example, yellow lines can be used for interstate highways and purple lines can be used for county highways. The incompatibility of these colors (no road can be both yellow and purple) is in accordance with the incompatibility of their referents no road can (ordinarily) be both an interstate highway and a county highway. If however, green is then introduced to indicate a toll road, then the incompatibility constraint will possibly be violated, since a county highway may be a toll road as well.

Although colors don't admit much granularity due to the impracticality of perceiving different similar shades of a single color, within degrees of freedom represented through other symbolic means, complex information can be filled out extensively without risk so long as the locatable features of the referents are all incompatible.

> Mountain marks pair with smooth texture as mutually incompatible, but syntactically significant, aspects of a map. Once mountain texture is on the menu, it is easy to add more textures for different kinds of land: alps, pied-

[4] Detail of Greater Los Angeles Freeway System Map reproduced by permission of Peter Dunn. Image from http://www.stonebrowndesign.com/

[5] OpenStreetMaps cartography is licensed under Creative Commons Attribution-ShareAlike 2.0. Image from http://www.openstreetmap.org/

> mont, hills, bumps, etc. Each of those textures is incompatible with the others, and what each represents is incompatible with what the others represent. Untextured, smooth areas are the zero value along this degree of freedom. Being smooth carries representational weight just as the marks do [21].

In practice, when symbolic schemes violate their incompatibility constraints, redundant representational strategies can sometimes prevent a critical failure of coverage from occurring. In Peter Dunn's beautiful redesign of the LA freeways system map (Figure 4), the Santa Monica Freeway (blue) briefly passes through and co-designates a short stretch of the Golden State Freeway (yellow). Dunn solves the incompatibility of the color designators by replacing the solid line of the Santa Monica Freeway with a dotted line for the portion of the two highways in which they overlap.

Similar ambiguity is frequently encountered in scores. "Hairpins," which indicate a change in volume, have an ambiguous meaning when they pass under rests (Figure 6). Although several incompatibilities seem to be at work here, the most promising way to explain the problem is that hairpins refer to an interpolation of sound intensity over the duration of one or more sounds. A hairpin under a rest may be syntactically sound but semantically flawed. Certain composers, notably Brian Ferneyhough, have adopted a dotted notation that clarifies this ambiguity. As in Dunn's map, a redundant symbol (the dotted hairpin) is incorporated into the map only in case the primary symbol encounters incompatible features.

Dunn's use of color gradients to symbolize transition points between symbols within a single degree of freedom (represented by colored solid lines—highways) is particularly notable. Color and abstraction of vectors is a key to a design that illuminates opaque aspects of a more traditional, highly isomorphic OpenStreetMap visualisation (Figure 5). In Dunn's map a gradient indicates an exit or on-ramp whereas a mitered join indicates an overpass or underpass. This layering of representational strategies through color, shape, and spatial organisation is a key to the ability of maps to represent numerous dimensions of information in an abstract gestalt unit.

Whereas traditional notational strategies for "extended techniques" (atypical means of producing sound on an instrument) conventionally rely on introducing different symbols along different degrees of freedom for each new specified extended technique, a more idiomatic cartographic representation would represent incompatible extended techniques as symbols featuring incompatibility constraints.

For example, a traditional symbolic notation for *col legno tratto* (bowing a string instrument with the wood of the bow) potentially yields an impossible map when it is erroneously layered with a symbolic notation for scratch tone (playing with extreme pressure of the bow hair) (Figure 7). Since playing heavily with the bow hair is largely incompatible with bowing with the wood of the bow, these two

[7] Score excerpt from *this will be changed and made solid II* used by permission of the composer.

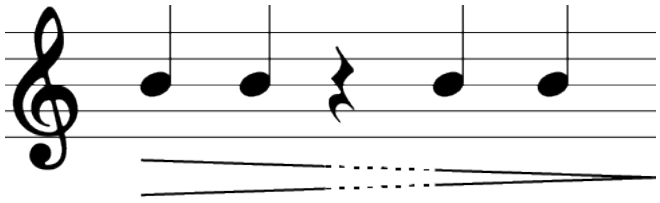

Figure 6. Hairpin with dotted-line notation to resolve ambiguity created by passing under rests.

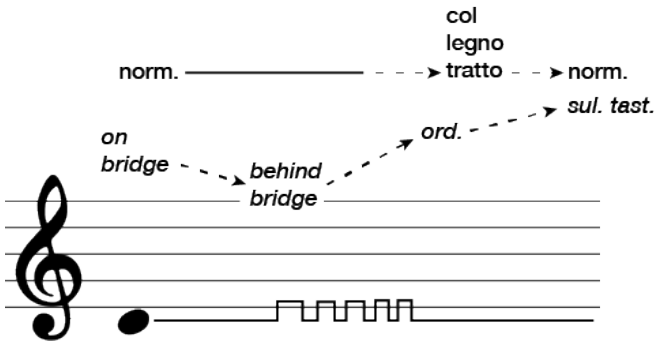

Figure 7. Traditional notation of extended techniques often exhibits poorly formed incompatibility constraints that allow for impossible layering effects. Too many types of representation, including words, symbols, and spatial distribution, are used simultaneously leading to difficulties in viewing the notation as a gestalt.

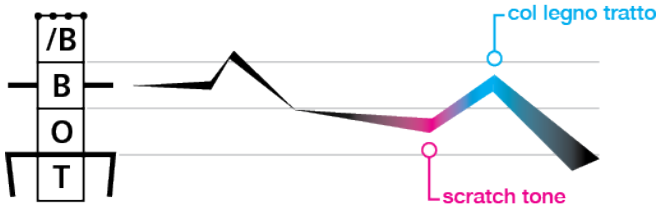

Figure 8. A hypothetical notation with well-formed incompatibility constraints and a more isomorphic graphic approach.

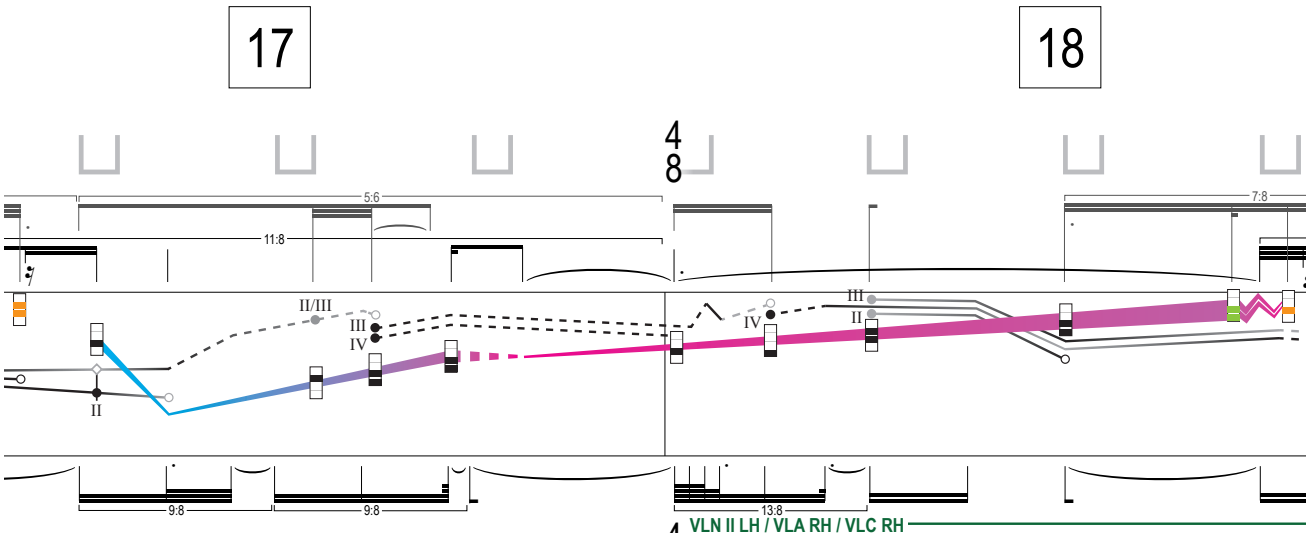

Figure 9. An excerpt from *this will be changed and made solid II* for string quartet by James Bean. Color represents bow placement; colored-line thickness represents bow pressure; colored-lined vector represents bow movements; vertical, striated rectangles represent the string to be bowed, while the height of these "string indicator boxes" represents the maximum bow pressure; black or dashed vectors and filled or open circles/diamonds represent finger movements along the strings with the indicated amounts of pressure; rhythms above the staff indicate temporal placement of right-hand information while rhythms beneath the staff are for the left-hand. [7]

techniques should be represented through an "incompatibility constraint" in the score. In Figure 7 the extensive layering of different symbolic strata is difficult to perceive as a gestalt, and it is nearly impossible to tell how and when the scratch tone becomes *col legno tratto* or normal tone. A separate degree of freedom has been introduced for pitch, bow placement, and two different degrees of freedom are used to notate bow timbre. Yet another degree would be required to show dynamics, although this too might be incompatible with the overpressure which is difficult to execute softly.

Figure 8 shows the notation of a similar passage to that shown in Figure 7. However, incompatibility-constrained symbols (the colors cyan and magenta) are used to show *col legno tratto* and scratch tone respectively. As in Dunn's map, the use of semantic incompatibility to show syntactically incompatible features entirely clarifies the smooth transition between the timbrel techniques. Additionally, to aid in the formation of a gestalt representation, line thickness is here used to show bow pressure and vertical spatial distribution (isomorphic to the body of a string instrument) is used to notate bow placement. [6]

An example of interesting and well-designed spatial-temporal scheme is found in *this will be changed and made solid II* for string quartet by James Bean (Figure 8). Written in 2012 and relying on extensive unconventional graphics created in the Adobe Illustrator software environment, the score is an interesting and complex type of tablature. By notating only the movements and actions of the right and left hands, the score remains consistent in its field of reference. There is no need to switch between perceptual descriptors (dynamics, expressive bowing, etc.) and physical actions (the absolute pressure of the bow on the string, its placement on the instrument, the movement of the left hand fingers, etc.). At the same time, by placing all tactile information in the center of the staff and reserving the extremities for rhythmic notation, the eye is better able to track a melodic gesture as a single multidimensional contour.

Bean's score takes the form of several maps layered on top of one another within the same representational space. For vectors relating to the movement of the left hand, the top of the staff is to be considered the bridge while the bottom of the staff symbolizes the nut. For vectors relating to the movement of the right hand, the top of the staff represents the frog of the bow while the bottom of the staff is the tip. Further research should consider what effect this multilayered scheme might have on map perception, coverage, and incompatibility of degrees of freedom and whether there are examples of multilayered representations in other fields.

4. CONCLUSIONS

The practice of score-making in the 20th and 21st centuries has become so varied and complex that it is impractical for a single theory of representation to encompass all cases. Certainly there are scores, such as Karlheinz Stockhausen's *Aus den sieben Tagen,* that operate on an entirely sentential basis. It is harder to think of scores that very closely approximate Goodmanian notation, but piano rolls—the long perforated paper scrolls that are the con-

[6] Although inspired by Helmut Lachenmann's "bridge clef," the spatial depiction of the string instrument is here modified to be visually discrete in its vertical layout. The use of Frutiger typeface, a high-visibility font designed for signage in Charles de Gaulle Airport, is also an innovation borrowed from "wayfinding" design. Space does not permit a discussion of the interesting parallels between wayfinding and cartographic modes of representation. The letter abbreviations are inspired by a system of bow placement indications used by Timothy McCormack.

trol interface for a mechanical player piano—might count if they are in fact a kind of score.

In Cage's 1969 book *Notations*, Jean-Charles Franois finds a hint of the doubts Goodman sought to answer in *Languages of Art*.

> "As soon as there is a necessity to demonstrate unequivocally that there is something to show, one has to persuade oneself that there is something to be shown. Here we find an infinite nostalgia for an ancient world in which the question of representation would never have been asked or considered in the first place."

Notations, a collection of pages from scores in a wide variety of graphic styles, signaled a change in the traditionally held view of scores [22]. No longer were music scores to be regarded as the "crystal goblet," as Beatrice Warde famously said of good type in her 1955 essay ("The book typographer has the job of erecting a window between the reader inside the room and that landscape which is the author's words." [23]). Instead, *Notations* is premised on a very modern conception of the score as a multimedia, multimodel object whose relation to the musical work is complex, often abstract or indirect, and highly conventionalized but nevertheless capable of expressing complex relations between objects and events in space and time that would not easily be conveyed in sentential form.

The increasing adoption of new software paradigms for notation combined with highly specific, systematised, or graphic notations developed by composers such as Timothy McCormack, Aaron Cassidy, and Cat Hope suggests the importance of developing a philosophical approach that can better analyze multiple modes of representation as functioning simultaneously within a temporally and spatially isomorphic representation. A cartographic theory of scores gets us a little bit closer to untangling that complexity.

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