INTERPRETING CHORD DISTANCES OF TONAL PITCH SPACE: A CORRELATION STUDY WITH CORPUS-BASED CHORD PROGRESSION FREQUENCIES

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ABSTRACT

This paper investigates the musical meaning of chord distances computed by Tonal Pitch Space (TPS) , a theory quantifying numeric distance between chords. TPS is a useful theory, but the musical meaning of its proposed chord distance remains unclear. Prior research has not attempted to verify this meaning. We propose two interpretations for chord distance: a value related to the typicality of chord progressions (i.e., ”transition” hypothesis) and a value related to the similarity of the chords (i.e., ”substitutability” hypothesis). We explore if TPS distances represent chord ”transition” or ”substitutability” by relating them with chord progression trigram frequencies from the MidiCaps corpus. Two ratios are calculated: occurrence ratio (simple frequency) and relative occurrence ratio (relative to the most frequent chord). The results refuted both the ”transition” hypothesis and the ”substitutability” hypothesis. However, during the validation of the transition hypothesis, a bell-curve-shaped relationship was observed between chord distance and chord occurrence frequency. When considered in light of the Wundt curve in experimental psychology, this suggests that chord distance may be functioning as a model for inferring chord novelty. No significant differences were observed between popular music and jazz.

1. INTRODUCTION

In popular music and jazz, composition and analysis often revolve around chords, making it essential for computers to understand chord progression theory to achieve automatic composition and music recommendation. Realizing computational chord progression theories would enable, for example, the identification of characteristic chord progressions from user-liked songs to recommend similar music based on those progressions.

Tonal Pitch Space (TPS)[1] is a convincing theory to quantify numeric distance between two chords. TPS is a prominent computational music theory proposed by Fred Lerdahl in 2001, serving as a complementary theory to the Generative Theory of Tonal Music (GTTM). Among computational music theories, TPS is unique in its specific focus on chords (i.e., harmony analysis).

However, it remains unclear what the chord distance computed by TPS specifically signifies as a musical theoretical value, and whether this chord distance should be considered an interval scale or an ordinal scale. Even in Lerdahl's original work[1], there is no clear description regarding the meaning of chord distance or the type of scale it represents. Although several studies have attempted to improve or extend TPS[2, 3, 4], no prior research has thoroughly verified and examined the meaning indicated by chord distance.

Chord distance appears to have two possible interpretations. One perspective views chord distance as a measure of the typicality of a chord progression, where smaller distances indicate more typical or frequently occurring progressions. We call it ”transition” hypothesis. The other perspective interprets chord distance as a measure of chord substitutability, suggesting that chord progressions with smaller distances share common harmonic functions, implying that chords can be interchanged while maintaining musical coherence. We call it ”substitutability” hypothesis.

We aim to re-examine the meaning and evaluate the validity of chord distances calculated by TPS by investigating their relation with trigram frequencies of chord progressions obtained from a chord progression corpus. In this paper, a set of chord trigrams sharing the same Head and Tail chords is defined as a single context.

2. TONAL PITCH SPACE

TPS is the quantitative harmony analysis method. In TPS, the distance between two chords (x and y) is defined as the sum of distances in tonal space, harmonic function space, and basic space as

$$\delta(x, y) = \mathrm{region}(x, y) + \mathrm{chord}(x, y) + \mathrm{basic\_space}(x, y).$$

The $\mathrm{region}(x, y)$ represents distance in tonal space. The basic method involves selecting the shorter distance on the circle of fifths (either clockwise or counter-clockwise) required to shift from the key of chord x to the key of chord y (Figure 1). For minor keys, transposition to their relative major is first considered.

The $\mathrm{chord}(x, y)$ represents distance in harmonic function space. Specifically, it involves converting chords x and y

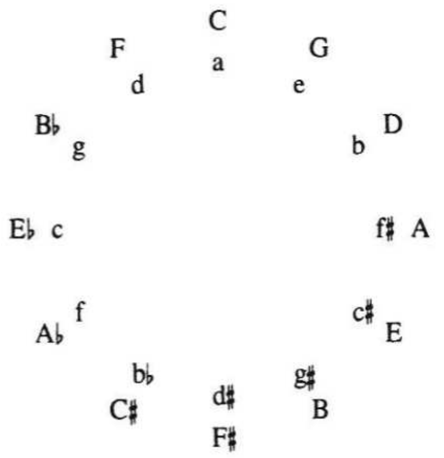


Figure 1: Circle of fifths (Kellner's regional circle)[1, p.47]

into their diatonic degree notation and then selecting the shorter distance on the common tone circle (either clockwise or counter-clockwise) to move between them (Figure 2).

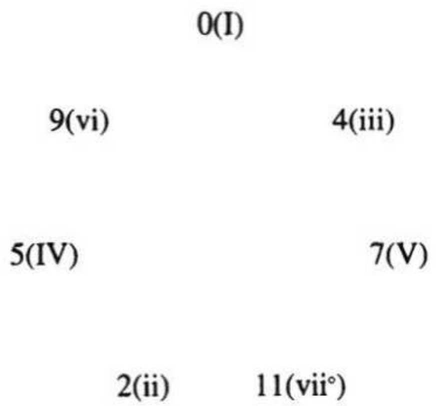


Figure 2: Common tone circle[1, p.56]

The $\mathrm{basic\_space}(x, y)$ function represents the weighted change in chord tones, categorized by their harmonic function. The basic space is a particularly important concept in TPS. It is structured as a 12x5 matrix, where the horizontal axis represents the 12 pitch classes from C to B, and the vertical axis represents five levels denoting the function of each chord tone (Table 1). For minor keys, while there are three types of scale tones-—natural minor, harmonic minor, and melodic minor-—the natural minor scale is used for creating the basic space. Figure 3 shows an example of

Table 1: Five Levels in the Basic Space

Level a:	Tonic (1st) tone
Level b:	Dominant (5th) tone
Level c:	Sub dominant (4th) and other chord tone
Level d:	Diatonic tone (Major or Natural Minor)
Level e:	Chromatic tone

the basic space representation for Dm7 in C major key and Am in A minor key. In TPS, chord tones must always be diatonic tones, meaning that TPS can only represent diatonic chords. The basic space distance is defined as the count of components that have been moved, increased, or decreased when comparing the basic space of the two chords.

Figure 4 shows an example of calculating chord distance using TPS, specifically for chord x as Dm in C major key and chord y as G7 in C major key.

3. OUR APPROACH

3.1 Core question

Existing research (e.g. [2, 4]) interpreted TPS-computed chord distances as measures of distance of chord transition propensity; however, we questioned this interpretation. If chord distance represents the inverse concept of transition typicality (referred to as the "transition" hypothesis) (Figure 5a), then it would be natural for the distance of the chord progression $x \to y$ to differ from that of $y \to x$. However, the chord distance calculated by TPS is symmetrical (i.e., $\delta(x, y) = \delta(y, x)$). If chord distance represents the inverse concept of chord substitutability (referred to as the "substitutability" hypothesis), then the values of $\delta(x, y)$ and $\delta(y, x)$ should be identical.

The core question of this paper is to ascertain whether the "transition" hypothesis or the "substitutability" hypothesis correctly explains the chord distance calculated by TPS.

3.2 Hypothesis

We propose two hypotheses in response to the aforementioned core question.

If the "transition" hypothesis is correct, then more typical chord progressions should yield smaller chord distances, resulting in a monotonically decreasing relationship between a chord's contextual likelihood of appearance and its chord distance as shown in Figure 6a.

Alternatively, if the "substitutability" hypothesis is correct, then more compatible chords should yield smaller chord distances, resulting in a monotonically decreasing relationship between a chord's substitutability within a context and its chord distance as shown in Figure 6b.

3.3 Methodology for Validation

We validate the core question by investigating the relationship between the occurrence frequencies of chord progressions derived from a chord progression corpus and the chord distances calculated by TPS. For our experiments, the MidiCaps music corpus, developed by Jan Melechovsky et al.[5, 6], was utilized, primarily due to its extensive song collection and comprehensive inclusion of chord names, genre, and key information.

To facilitate the subsequent explanation, we define the set of diatonic triads and seventh chords as

$$\mathbb{D} = \{\text{`C'}, \text{`Dm'}, \text{`Em'}, \cdots, \text{`Am'}, \text{`Bm}^{-5}\text{'}, \\ \text{`Cmaj7'}, \text{`Dm7'}, \text{`Em7'}, \cdots, \text{`Am7'}, \text{`Bm7}^{-5}\text{'}\}.$$

	C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B
Level a			D									
Level b			D							A		
Level c	C		D			F				A		
Level d	C		D		E	F		G		A		B
Level e	C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B

(a) Dm7 in C major Key

	C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B
Level a										A		
Level b					E					A		
Level c	C				E					A		
Level d	C		D		E	F		G		A		B
Level e	C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B

(b) Am in A minor Key

Figure 3: Basic space

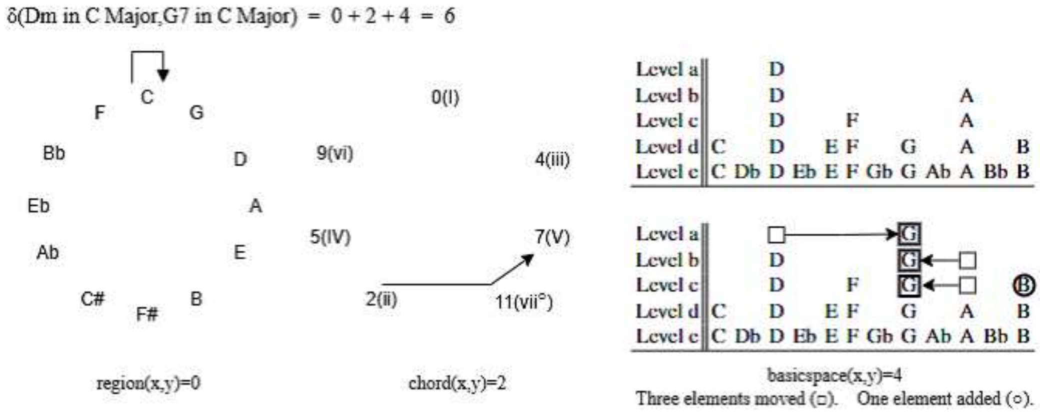


Figure 4: Calculation sample of chords distance by TPS

And we define a chord progression trigram, considering trigrams with identical C_{head} and C_{tail} chords to be a single context:

$$\text{trigram} \stackrel{\text{def}}{=} [C_{head} \rightarrow X \rightarrow C_{tail}] \quad (C_{head}, X, C_{tail} \in \mathbb{D}) .$$

For each context, we aggregate the occurrence frequencies of the chord X within the corpus(Figure 7). When described as a conditional probability, the occurrence frequency is represented by

$$P\left(X = x_i \mid C_{head} = c_1, C_{tail} = c_2\right) \quad (1 \leq i \leq N) ,$$

where N denotes the number of distinct chord X types appearing in a context.

And we define x_{max} as the chord X with the maximum occurrence probability as

$$x_{max} \stackrel{\text{def}}{=} \underset{x_i}{\text{argmax}} P\left(X = x_i \mid C_{head} = c_1, C_{tail} = c_2\right) \quad (1 \leq i \leq N) .$$

From the obtained occurrence frequencies, we calculate two metrics: occurrence ratio and relative occurrence ratio. These two types of ratios differ in their denominator, which forms the basis of their calculation.

The occurrence ratio (OR) is the simple appearance ratio of a chord X, calculated with conditional probability:

$$\text{OR}(c_1, x, c_2) = P\left(X = x \mid C_{head} = c_1, C_{tail} = c_2\right) .$$

As an example represented in Table 2, with the occurrence count of context "C-X-G7" (457) as the denominator, the ratio where X is F is calculated as $251/457 = 54.92\%$, and the ratio where X is Dm7 is calculated as $90/457 = 16.69\%$.

The relative occurrence ratio (ROR) is the appearance ratio of other chords X, calculated using the occurrence count of x_{max} (the most frequent chord X) within the same context as the denominator:

$$\text{ROR}(c_1, x, c_2) = \frac{P\left(X = x \mid C_{head} = c_1, C_{tail} = c_2\right)}{P\left(X = x_{max} \mid C_{head} = c_1, C_{tail} = c_2\right)} .$$

As an example represented in Table 2, with the occurrence count of most frequent chord x_{max} F in context "C-X-G7" (251)as the denominator, the ratio where X is Dm7 is calculated as $90/251 = 35.86\%$, and the ratio where X is Dm is calculated as $52/251 = 20.72\%$.

3.4 Relation Between Two Occurrence Ratios and Chord Distance

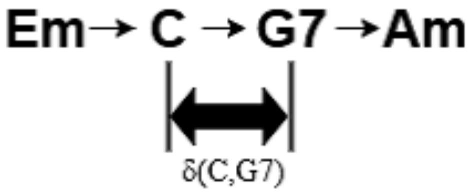
In the following equations, the symbol $\leftrightsquigarrow$ indicates a relationship (not limited to linear) between the left and right sides.

If the "transition" hypothesis holds true—that is, if TPS's chord distance represents the chord transition propensity (CD_{tr})—then some relation should be observable between CD_{tr} and the OR as

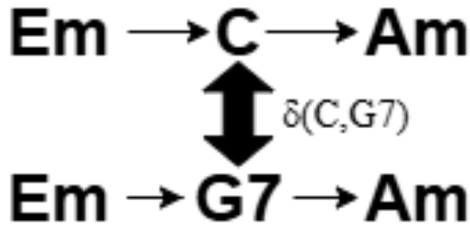
$$\text{OR}(c_1, x, c_2) \leftrightsquigarrow \text{CD}_{tr} .$$

CD_{tr} is defined as the sum of the chord distance from C_{head} to X and the chord distance from X to C_{tail} as

$$\text{CD}_{tr}(c_1, x, c_2) \stackrel{\text{def}}{=} \delta\left(c_1, x\right) + \delta\left(x, c_2\right) .$$

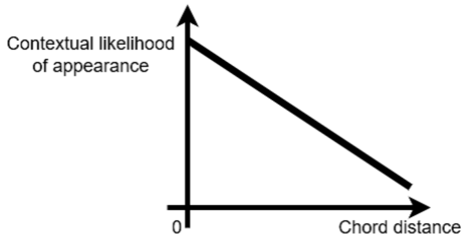


(a) Pattern A: TPS-computed chord distances is measures of chord transition propensity.

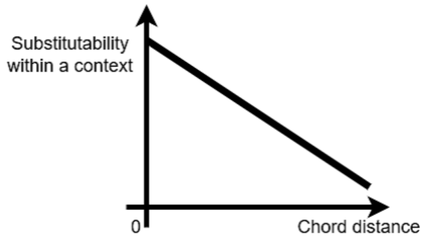


(b) Pattern B: TPS-computed chord distances is measures of chord "substitutability" propensity.

Figure 5: Two Interpretations of Chord Distance



(a) Relationship between a chord's contextual likelihood of appearance and its chord distance



(b) Relationship between a chord's substitutability within a context and its chord distance

Figure 6: Hypothesis

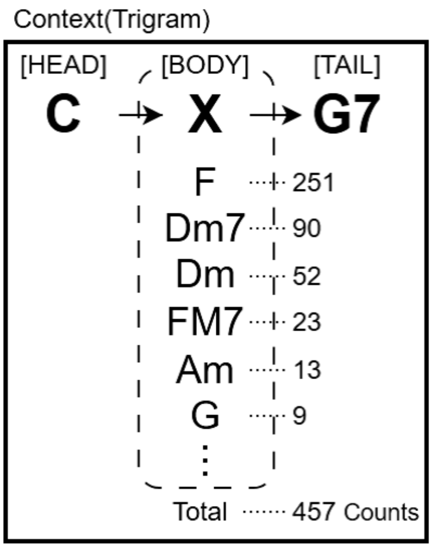


Figure 7: Sample of occurrence frequencies

If the "substitutability" hypothesis holds true-—that is, if TPS's chord distance represents the substitution propensity (CD_{sb})-—then some relation should be observable between CD_{sb} and the ROR as

$$\mathrm{ROR}(c_1, x, c_2) \leftrightsquigarrow \mathrm{CD}_{sb}(c_1, x, c_2).$$

CD_{sb} is defined as the chord distance from X to x_{max} as

$$\mathrm{CD}_{sb}(c_1, x, c_2) \stackrel{\mathrm{def}}{=} \delta\left(x, x_{max}\right).$$

4. EXPERIMENTS

4.1 Procedure

4.1.1 Trigram Extraction

From the MidiCaps chord progression data, we selected only songs categorized as Jazz or Pop, then transposed major keys to C Major and minor keys to A minor before extracting the chord progression trigrams. We used the key data provided in the corpus for the original keys before transposition.

4.1.2 Trigram Frequency Aggregation

Trigrams are extracted and aggregated from the data prepared in §4.1.1. However, trigrams containing non-diatonic chords are excluded, as TPS can only process diatonic chords. Trigrams with repeated consecutive chords are also excluded. First, we fix the leading (C_{head}) and trailing (C_{tail}) chords of a trigram as the context, then aggregate the occurrence frequencies of the central (X) chord. We then calculate the OR and ROR.

4.1.3 Chord Distance Calculation and Relation Analysis

For the trigrams obtained in §4.1.2, the chord distances will be calculated using TPS according to the algorithm detailed in §2. We will calculate two types of chord distances: CD_{tr} & CD_{sb}

From the results obtained through the above procedures, a scatter plot will be generated to investigate the relation.

4.2 Results

To test the "transition" hypothesis, we generated a scatter plot showing the relation between CD_{tr} and OR; however,

Table 2: Occurrence ratio / Relative occurrence ratio

Context	Total		$x_1 = x_{max}$	x_2	x_3	x_4	x_5	x_6	$\cdots$
			F	Dm7	Dm	FM7	Am	G	$\cdots$
C-X-G7	457	Counts	251	90	52	23	13	9	$\cdots$
		occurrence ratio (OR)[%]	54.92	16.69	11.38	5.03	2.84	1.97	$\cdots$
		relative occurrence ratio (ROR)[%]	100	35.86	20.72	9.16	5.18	3.59	$\cdots$

contrary to the hypothesis, no monotonically decreasing relationship was observed, but rather a bell-shaped distribution with chord distances clustering around 13-14. Furthermore, a comparative analysis conducted by performing the same analysis on Jazz and Pop genres revealed no discernible differences between the two. Figure 8 presents the distribution charts created for five representative contexts.

Next, to test the "substitutability" hypothesis, we generated a scatter plot showing the relation between CD_{sb} and ROR; while no clear relation was found, chords with a chord distance of 1 (i.e., differing only by the presence or absence of a 7th) were distinctly separated from all others. No differences were observed between Jazz and Pop in this analysis either. Figure 9 presents the distribution charts created for five representative contexts.

Figures 10 and 11 present detailed analysis results for two representative contexts.

Figure 10 illustrates the context where the chord progression moves from Am (C_{head}) to another chord (X) and then returns to Am (C_{tail}). The graph of CD_{tr} – RO exhibits a broad bell shape, indicating that a diverse range of chords can serve as the X. Similarly, the graph of CD_{sb} - ROR also shows a wide distribution.

Conversely, Figure 11 presents the context where the chord progression moves from Dm7 (C_{head}) to another chord (X) and then proceeds to C (C_{tail}). In this specific context, the chord X is almost exclusively G7 or G, representing a typical ii-V-I progression. The graph for CD_{tr} - OR displays an extremely narrow bell shape, indicating that the variety of chords that can function as the X is severely limited. Similarly, in the chord distance between CD_{sb} - ROR graph, only chords with a chord distance of 1 (i.e., those differing merely by the presence or absence of a 7th) are prominently used, again suggesting a very restricted range of chord choices.

4.3 Consideration

4.3.1 About Hypothesis Verification

Initially, the "substitutability" hypothesis is refuted due to the lack of a discernible relation between CD_{sb} and ROR.

A bell-shaped relationship was observed between CD_{tr} and OR, which, not being the monotonically decreasing relationship assumed by the "transition" hypothesis, suggests that the "transition" hypothesis is not correct.

Thus, it's evident that chord distance is not a simple metric that directly correlates with occurrence frequency derived from the chord corpus.

4.3.2 Interpretation of Chord Distance

While both the "substitutability" hypothesis and "transition" hypothesis were refuted, the observed bell-shaped relationship between CD_{tr} and OR is noteworthy, as theories in experimental psychology propose a similar bell-shaped (inverted U-shaped) relationship between stimulus complexity (or novelty) and pleasantness. This theory originates from the Wundt curve (Figure 12), proposed by psychologist Wilhelm Wundt, and has been subsequently supported by numerous researchers.

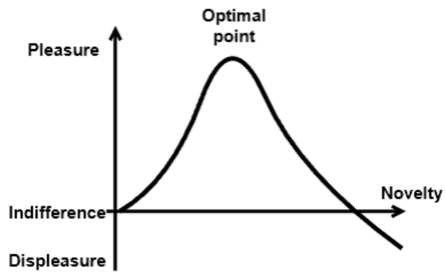


Figure 12: Wundt Curve

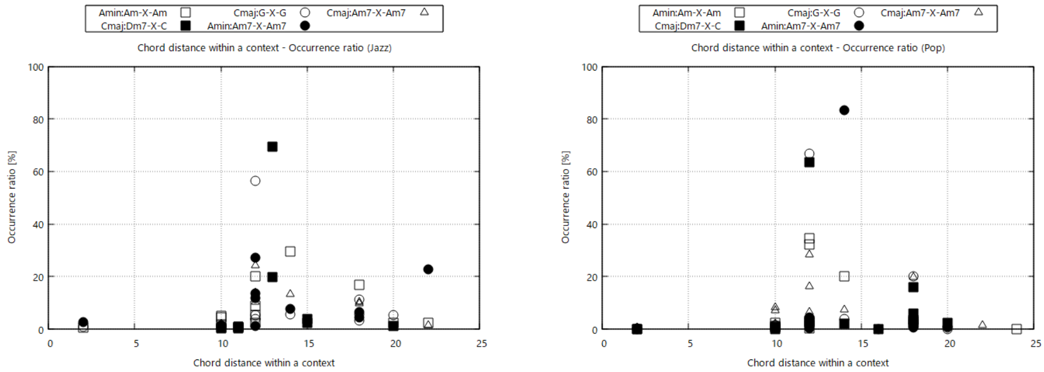
If considered in light of the Wundt curve, chord distance may serve as a model for inferring chord novelty; however, precisely defining chord novelty is challenging, making it difficult to immediately ascertain whether chord distance accurately infers it. Further investigations are necessary, incorporating insights from music psychology and cognitive theories.

5. CONCLUSION

In this paper, to verify whether the chord distance calculated by TPS indicates the chord transition propensity (CD_{tr}),that is, "transition" hypothesis, or the chord substitutability (CD_{sb}), that is, "substitutability" hypothesis, we calculated the relation between TPS-derived chord distances and occurrence frequencies obtained from a chord progression corpus using two distinct methods.

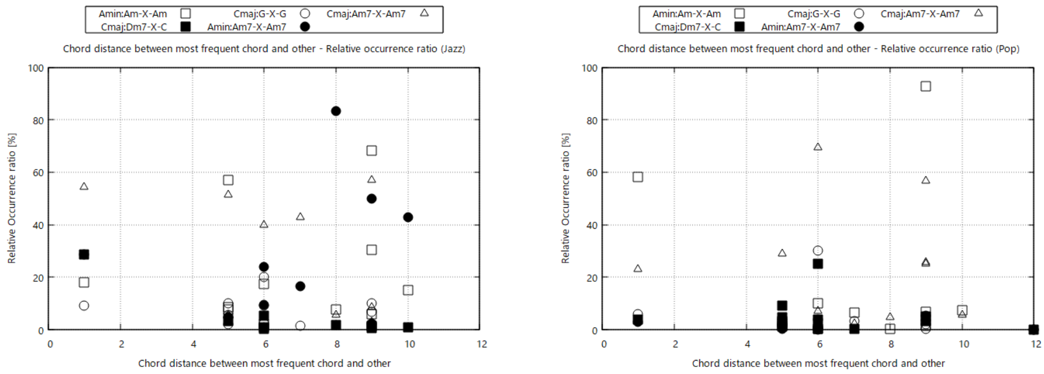
If the "transition" hypothesis holds true, then a scatter plot of CD_{tr} versus Occurrence ratio (OR) should reveal some relation; indeed, the results showed a tendency for chord distance to exhibit a bell-shaped distribution, primarily centered around 13-14. This result is analogous to the Wundt curve and suggests the possibility that chord distance functions as a model for inferring chord novelty.

If the "substitutability" hypothesis holds true, then a scatter plot of CD_{tr} versus Relative Occurrence ratio (ROR) should reveal some relation; although no clear relation was



(a) Jazz (b) Pop

Figure 8: A scatter plot showing the relation between CD_{tr} and OR



(a) Jazz (b) Pop

Figure 9: A scatter plot showing the relation between CD_{sb} and ROR

found in this scatter plot, chords with a chord distance of 1 (i.e., those differing only by the presence or absence of a 7th) were distinctly separated from all others.

In both results, no significant differences were observed between the Jazz and Pop genres.

Our future objectives are to perform verification using statistical models rather than just scatter plots, and to expand the analysis to non-diatonic chords utilizing the authors' proposed TPS-ExJ model.

Acknowledgments

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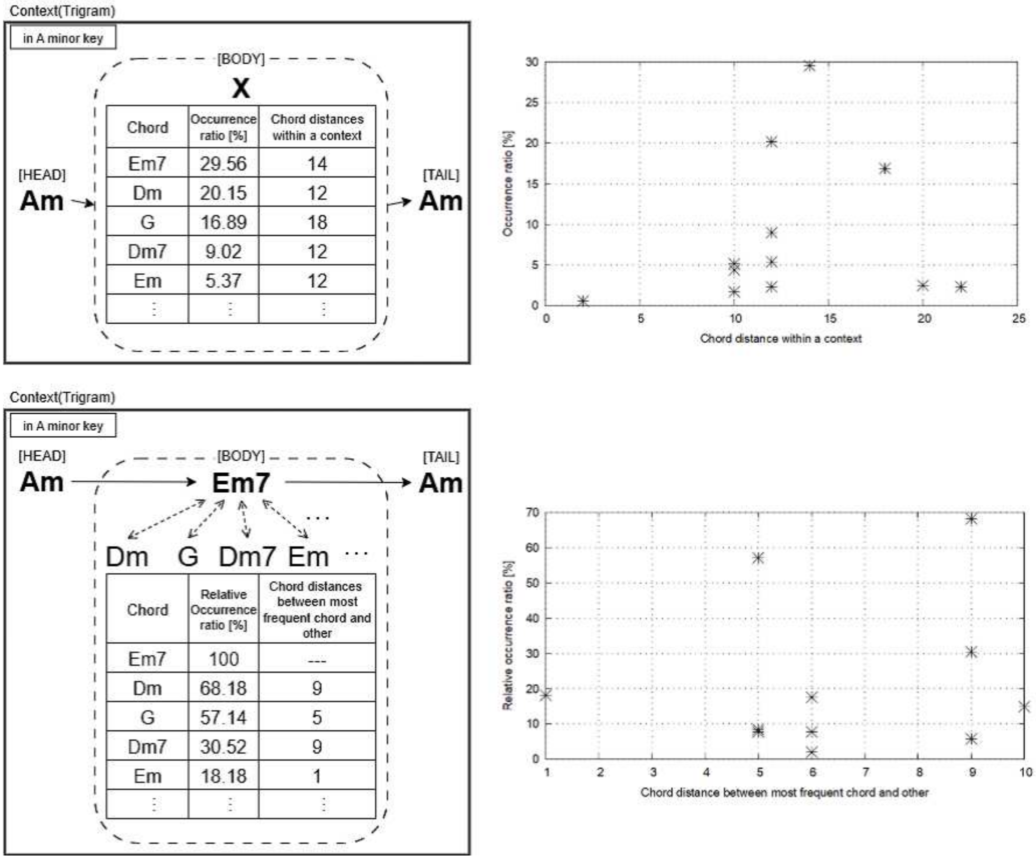

Figure 10: Result viualisation (Am-X-Am in A minor key (from genre 'Jazz'))

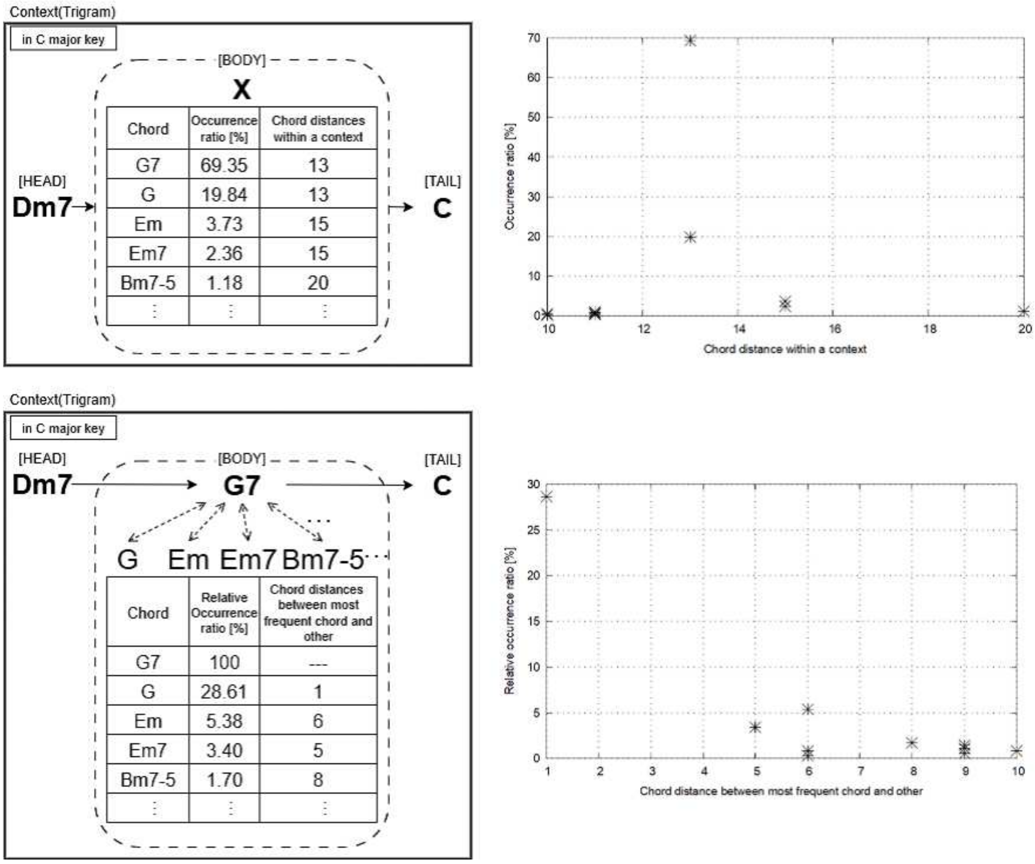

Figure 11: Result viualisation (Dm7-X-C in C major key (from genre 'Jazz')