

RATIOSCORE: A TEXT-BASED SYSTEM FOR JUST INTONATION

Julie Zhu
Stanford University
julie@ccrma.stanford.edu

Craig Stuart Sapp
CCARH/PHI, Stanford University
craig@ccrma.stanford.edu

ABSTRACT

Ratioscore is a text-based musical score representation system that expresses pitches as integer ratios, making it particularly useful for working with just intonation. We also provide open-source software to compile Ratioscores into Standard MIDI files as well as a website front-end that renders these MIDI files into MP3 files for online listening and downloading. Up to 15 voices/instruments can sound simultaneously with independent tuning control as well as independent glissandi in each voice within one octave of their starting pitch. Ratioscores are encoded using the Humdrum data format, which allows for basic data manipulation and transformation of the score, such as repeating segments of music, selecting between multiple timelines or sharing dynamics between different voices. We illustrate the use of Ratioscore as a prototyping system for composing a string quartet as a proof of concept. Ratioscore is particularly suited for algorithmic composition, both within and outside the traditional Western notation framework.

1. INTRODUCTION

The overlap between just intonation (JI) and the beauty of mathematics is substantial. Young composers inclined towards a kind of numerical perfection are increasingly drawn into the just intonation “cult.” In addition to more pieces composed with just intonation, theory and resources grow alongside to help composers navigate the just world.

Before Marc Sabat and Wolfgang Von Schweinitz developed accidental-based notation, the Extended Helmholtz-Ellis Just Intonation (HEJI) pitch notation, there were numerous ways to notate just pitches. A plus or minus could be added to a note to indicate a rough deviation [1], or a cent differential from the equal tempered pitches as described in [2], or simply the JI ratio with a specified fundamental. In 2004, Plainsound Music Edition published Sabat and Von Schweinitz’s catalogue of accidental groupings that indicate prime-groupings and thus, families of cent deviations from the equal temperament, which has become standard for JI notation [3].

JI playback has not been standardized. Max and Super-Collider, along with Thomas Nicholson’s web application can calculate JI harmonic spaces, but that’s only to collect

individual frequencies, preparatory abstractions [4]. Without bespoke programming, timelines of JI music are not easily produced. In the last two years, Dorico, a music notation software, in collaboration with Plainsound composers, developed a microtonal playback system that Sabat endorses [5]. He describes what many composers have resorted to doing before Dorico: making the scores look good in notation software, but accidentals are “simply visualizations: text attachments without actual functionality.” In other words, without a lot of manual adjustment, the notated music could not be played back accurately. Perhaps with the improvement of vibrato-less soundfonts (VSTs) such as NotePerformer, Dorico will become the go-to software for JI composers that are working with staff notation [6].

We propose Ratioscores as an additional resource, open-source and text-based, which offer easy access to tuning systems outside of traditional Western notation and equal-temperament for use with any General MIDI synthesizer. As with just-intonation tuning, pitches are given as fractions, which are ratios between the target pitch and a constant fundamental, which is notated by ‘1/1’ or ‘1.’

2. RATIOSCORE REPRESENTATION

Ratioscores are based on the Humdrum file format, which is a generalized two-dimensional digital score representation organized similarly to common Western music notation (CWMN) scores without system breaks that are rotated 90° clockwise so that time progresses downwards in the file.[7] Each row represents events that occur simultaneously, and columns (‘spines’ in Humdrum terminology) represent different data streams running in parallel, such as time, pitch and dynamics. Humdrum data can represent music in CWMN¹ but also allows creation of non-standard sequential digital descriptions of music, such as Ratioscores. Figure 1 gives an example of a simple Ratioscore containing a timeline and a single voice.² The same musical content is shown below the Ratioscore in CWMN using HEJI accidentals.

The first spine is headed with the text ‘**time’ and represents a timeline controlling the starting time and duration of notes found in ‘**ratio’ spines(s). For example, the second note (D5) occurs one second after the start of the music, and sustains for one second until the next note at two seconds from the start of the score. All data spines should end with the end-of-data marker ‘*-’ and each spine must be separated from each other by one or more tab characters

Copyright: ©2021 Julie Zhu et al. This is an open-access article distributed under the terms of the [Creative Commons Attribution 3.0 Unported License](https://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

¹ <https://verovio.humdrum.org>

² Audio versions of figures are available at <https://ratioscore.humdrum.org/doc/tenor2021>.

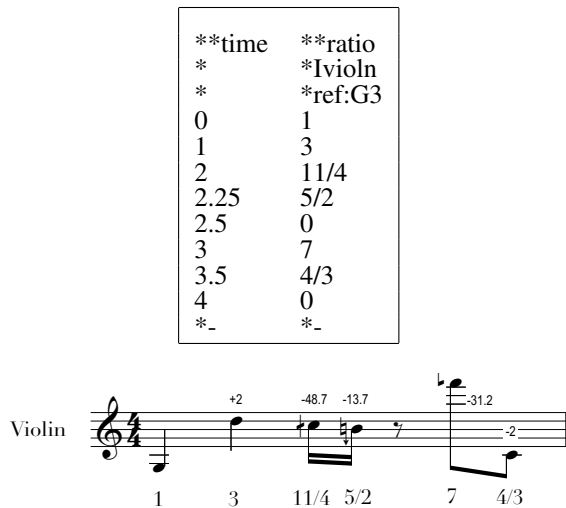


Figure 1. Sample music in Ratioscore and HEJI notation.

in a TSV (tab-separated-values) arrangement. Additional structural information about Ratioscores is available at the website, such as how to add metadata and comments to a score.³

2.1 Ratio spines

Each ratio spine represents a monophonic voice, with a new pitch ratio automatically turning off the previous note in the voice, or one can use '0' to silence a note without sounding a new one. Ratioscores are intended to be compiled into Standard MIDI files, so the number of ratio spines is limited to 15 since there are 16 MIDI channels available, minus channel 10 which is dedicated to percussion. Each ratio spine is mapped onto a separate MIDI channel, allowing independent tuning of notes in each voice with pitch-bend messages. Instruments can be specified, such as violin with the text '*Ivioln', or '*I#40' as a General MIDI instrument number.⁴

Pitches are indicated by integer ratios, either a single integer representing a harmonic or two integers separated by a slash. For example, '11/4' in Figure 1 is the 11th harmonic transposed down two octaves, 5/2 is the 5th harmonic transposed down an octave, and 4/3 is a just perfect fourth. Ratios can be adjusted by cent intervals, such as '3/2-1.955c', which is a just perfect fifth lowered by 1.955 cents to create an equal tempered (ET) fifth. Cents can also be interchanged with ratios for convenience, such as '701.955c' for a just perfect fifth.

The reference pitch for notes in a ratio spine is demonstrated in Figure 1 by '*ref:G3', where G3 is the G below middle C (assuming an equal temperament based on A-440), and all pitches for ratios are calculated from the reference pitch. If no reference pitch is given, then 'C4' (middle C) will be used, and the reference may be changed in a ratio spine at any time by indicating a new reference pitch. The reference pitch may also be tuned by adjusting a cent interval such as '*ref:G#3-12c', which means that

³ <https://ratioscore.humdrum.org>

⁴ See <https://ratioscore.humdrum.org/doc/instruments> for a list of other instrument codes and numbers.

the reference pitch is 12 cents flat of ET G#3. Reference pitches may also be described in Hertz by adding the letter 'z' after the frequency such as '*ref:261.63z' for 261.63 Hertz, which is equivalent to '*ref:C4'.

2.2 Exponent notation

Figure 2 demonstrates an alternate method for describing ratios with mathematical expressions. The clarinet part in this figure uses exponents to express stacks of just perfect fifths, while the violin part uses the same intervals multiplied out into a simplified rational number. The two parts play the same pitches, but the Pythagorean tuning is conveyed more clearly in the clarinet part. Currently only exponentiation is implemented, but other expressions to describe factorizations and roots may be added in the future.

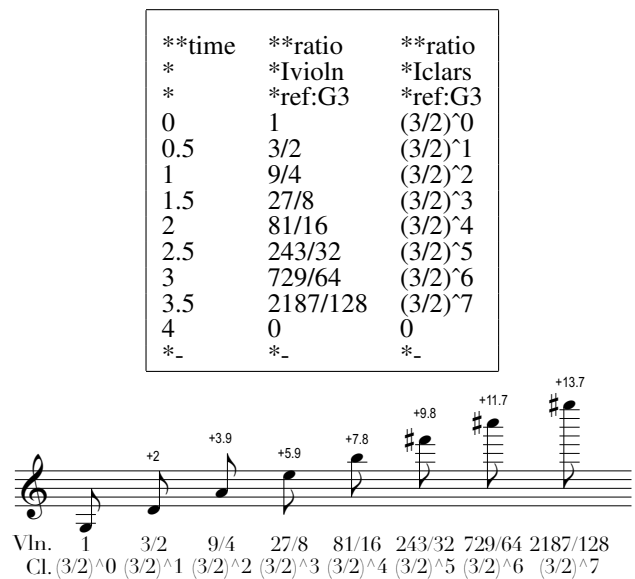


Figure 2. Ratioscore with rational number and equivalent exponential forms playing successive Pythagorean fifths.

2.3 Part alignment and barlines

The rhythm of all ratio spines is controlled by a single timeline spine, typically given in the first column of the score. To continue sustaining a note through an entry in the timeline, add a period/full-stop character ('.') in the sustaining note's column. Figure 3 has examples of these spacers where some parts are sustaining notes while others are attacking new ones. The figure also demonstrates addition of barlines in the score. These are indicated by equal signs (=) in each column followed by an optional measure/bar number. Barlines do not need to be evenly spaced in time and can split sustained notes, but if a Ratioscore is intended to represent CWMN, then time signatures such as '*M6/8' can be added to a ratio spine, and barlines can be placed to match the meter (Figure 3 includes a pickup beat).

2.4 Alternate timing methods

The controlling timeline in a Ratioscore can be expressed in several ways for convenience. The '**time' spine illus-

**time	**ratio	**ratio	**ratio	**ratio
*MM60	*Ivioln	*Ivioln	*Iviola	*Icello
*M4/4	*ref:G3+2c	*ref:D4+4c	*ref:C3	*ref:C2
0	1	.	.	6/5
0.5	.	1	6/5	.
=1	=1	=1	=1	=1
1	3	6/5	1	4/3
1.5	6/5	3	.	1
2	5	4/3	3	3/2
2.5	4/3	.	3/2	3
3	7	3/2	5	9/8
3.5	.	7	9/8	5
4	9	.	7	7/5
4.5	9/8	9	.	7
=2	=2	=2	=2	=2
5	11	7/5	9	9/7
5.5	7/5	11	9/7	.
6	13	9/7	.	13/11
6.5	9/7	13	13/11	11
7	15	13/11	13	.
7.5	13/11	15	.	13
=3	=3	=3	=3	=3
9
*_	*_	*_	*_	*_

Figure 3. Ratioscore with rational numbers and equivalent exponential forms.

trated in Figure 1 expresses time in absolute seconds from the start of the music. Additional descriptions of time are given in this section.⁵ In all cases, tempo adjustments can be made by adding a tempo indication in the form ‘*MM#’ where ‘#’ is a floating-point number giving the ‘beat’ count (seconds) per minute. The default tempo in timeline types is 60 beats per minute (bpm). Multiple timelines can be stored in a Ratioscore, but only the leftmost one will be used.

All timelines other than ‘**recip’ can use floating-point time values as well as rational numbers and mixed fractions, so ‘1.5’, ‘3/2’, and ‘1+1/2’ are all equivalent time values. Describing times with fractions allows tuplets to be easier to read from the scores, while floating-point values are useful for describing physical times. The ‘**recip’ system is a fractional delta-time system.

⁵ More details can be found at <https://ratioscore.humdrum.org/doc/timeline>.

2.4.1 Delta timings

Figure 4 demonstrates ‘delta times’ being used to represent the same timing information given as absolute timings in Figure 3. With delta times, time values are given in terms of the duration from the current line to the next. Expressing time in this format is similar to timings in MIDI files, and it allows for copy-and-pasting lines of the score to repeat musical content. In Figure 4, notice that the tempo is set to 120 bpm, which causes the delta times of ‘1’ to be equivalent to 0.5 seconds.

**dtime	**ratio	**ratio	**ratio	**ratio
*MM120	*Ivioln	*Ivioln	*Iviola	*Icello
*M4/4	*ref:G3+2c	*ref:D4+4c	*ref:C3	*ref:C2
1	1	.	.	6/5
1	.	1	6/5	.
=1	=1	=1	=1	=1
1	3	6/5	1	4/3
1	6/5	3	.	1
1	5	4/3	3	3/2
1	4/3	.	3/2	3
1	7	3/2	5	9/8
1	.	7	9/8	5
1	9	.	7	7/5
1	9/8	9	.	7
=2	=2	=2	=2	=2
1	11	7/5	9	9/7
1	7/5	11	9/7	.
1	13	9/7	.	13/11
1	9/7	13	13/11	11
1	15	13/11	13	.
3	13/11	15	.	13
=3	=3	=3	=3	=3
*_	*_	*_	*_	*_

Figure 4. Ratioscore with delta timing. Each line except the last one has a duration of 0.5 seconds.

2.4.2 Reciprocal timings

Ratioscore also allows reciprocal timings, which describe time in terms of divisions of a whole note. Numbers are expressed in terms of the reciprocal of the divisions, such as ‘4’ for a quarter note since it divides the whole note into four equal parts. Augmentation dots are represented by adding a dot after the rhythmic value, such as ‘8.’ for a dotted eighth note. Tuplets function in the same manner, with ‘3’ meaning a triplet half note, and ‘20’ being a quintuplet sixteenth note. More complex ratios that are not integer divisions of the whole note are represented by placing a percent sign between the two fractional parts of the number, such as ‘3%2’ which represents a triplet whole note since a triplet whole is 2/3rds of a regular whole note). Figure 5 gives an example of this timing system using the ‘**recip’ data type for the time column.

Tempo markings in *recip* spines describe the rate of quarter notes (‘4’), while in other timelines the tempo marking affects the duration of a second.

2.4.3 Millisecond timings

The data types ‘**ms’ and ‘**dms’ are similar to ‘**time’ and ‘**dtime’ respectively, but integers represent milliseconds rather than seconds.

**recip	**ratio
*M60	*Ivioln
*	*ref:G3
4	1
4	3
16	11/4
16	5/2
*MM44	*
8	0
8	7
8	4/3
*_	*_

Figure 5. Time described using musical rhythms (4:quarter note, 8:eighth note, 16:sixteenth note).

2.5 Glissandi

Glissandi are represented in Ratioscores by the letter ‘H’ attached to the starting note of the glissando and ‘h’ on the ending note as illustrated in Figure 6. Glissandi may extend up to one octave above/below the starting note, but no further due to the pitch-bend depth limits in MIDI. The Ratioscore conversion software analyzes the ranges of glissandi in each ratio spine and sets the necessary pitch-bend depth.

Intermediate notes within a glissando can be either sustained or attacked. Adding an underscore character will prevent a note from re-attacking during a glissando. The underscore character can also function as a tied note if the previous pitch matches the same note coming after it with an underscore marker. This is useful for encoding tied notes when the Ratioscore more closely represent a CWMN score.

The default update rate for pitch bends within a glissando is 50 milliseconds. For fast glissandi with large intervals, this may cause the discrete pitch-bend adjustments to become audible. In such cases the pitch-bend update rate can be manual adjusted, such as setting it to 10 milliseconds with the text ‘*grate:10’ (meaning ‘glissando rate’).⁶

2.6 Dynamics

Dynamics can be controlled through attack velocity and volume continuous controllers in MIDI files. Ratioscores allow separate control of these two types of loudness with ‘**vel’ and ‘**vol’ spines to the right of ratio spines that they apply to as illustrated in Figure 7.⁷ If a dynamics spine is placed to the left of all ratio spines, then it will apply to all ratio spines unless a ratio spine has its own dynamics spine.

Dynamics can either be MIDI-like numbers from 1 though 127, or they can be symbolic musical values such as ‘mf’ or ‘pp’. Symbolic values can be assigned specific MIDI numeric values as illustrated in the score, where ‘*pp:10’ means to convert ‘pp’ to the MIDI attack velocity 10. The ‘p’ and ‘f’ dynamics will use the default mappings since no explicit mapping are given for them in this example. Crescendi are the pair of characters ‘< [’ which are placed

⁶ See <https://ratioscore.humdrum.org/doc/glissandos> for more information.

⁷ See <https://ratioscore.humdrum.org/doc/dynamics>.

**time	**ratio	**ratio
*	*Ivioln	*Iclars
*	*	*grate:10
*	*ref:C4	*ref:C4
0	1H	0
2	_3/2	3/2
4	_1h	_3/2H
4.5	0	.
5	.	_1
6	.	.
6.5	3/2	_6/5
7	1H	.
8	_5/4h	5/4
9	.	_1h
10	0	1H
11	3H	_2h
12	_2h	.
13	0	0
*_	*_	*_



Figure 6. Sample score with glissandi

at the starting/ending times of the crescendi, and ‘>]’ are used for decrescendi. The ending bracket character can be omitted if a crescendo ends at a dynamic mark.

3. MIDI TUNING AND TIME QUANTIZATION

We provide software to convert Ratioscores into the Standard MIDI File format to realize just intonation.⁸ Typically the temperament of notes is not specified in MIDI data but rather is set on the synthesizer for playback of MIDI files.[8] There is no standard way of setting the temperament in a MIDI file, but synthesizer-specific temperaments can be encoded as system-exclusive messages.[9] However, these messages will only be understood by a particular synthesizer and will be ignored by others. We wanted an easy way to use any synthesizer, so note tuning

⁸ <https://github.com/craigsapp/ratioscore>

**time	**ratio	**vel
*	*Ivioln	*pp:10
*	*ref:G3	*
0	1	p
1	3	<
2	11/4	.
2.25	5/2	.
2.5	0	f >
3	7	.
3.5	4/3	.
4	0	pp
*_	*_	*_

Figure 7. Sample score with dynamics

The image shows four staves of musical notation for Cello, Viola, Vln. II, and Vln. I. Each staff contains notes with fingerings (Roman numerals I-IV) and ratios. The Cello staff starts with a fundamental C2 note (ratio 1) and includes notes with ratios 2, 9/4, 3, 4, 9/2, 6, 7, and 63/4. The Viola staff starts with a note at ratio 2 and includes notes with ratios 3, 7/2, 4, 6, 7, 8, 9, 12, and 14. The Vln. II staff starts with a note at ratio 3 and includes notes with ratios 27/8, 4, 9/2, 6, 27/4, 8, 9, 12, 27/2, 16, and 18. The Vln. I staff starts with a note at ratio 7/2 and includes notes with ratios 63/16, 3, 7, 63/8, 9, 21/2, 12, 27/2, 14, 63/4, 18, 21, and 27. The notes are represented by half notes on a five-line staff, with some notes having accidentals (flats) and some having circled fingerings.

Figure 8. Pitch space for *The Answers* based on the fundamental C2, the lowest note of the cello. Sixteen notes (not necessarily unique pitch, or ratio) for each viol.

The Answers

J.ZHU (2020)

Violin I

Violin II

Viola

Violoncello

$\text{♩} = 60$

p *pp* *mp* *mf* *p*

p *pp* *mp* *p* *mf* *p* *mp* *pp*

p *ppp*

p *ppp* *fp*

III IV I II I III

III IV II I

III m.s.t. ord.

IV

**time	**ratio	**ratio	**ratio	**ratio
*	*Icello	*Iviola	*Ivioln	*Ivioln
*	*ref:C2	*ref:C2	*ref:C2	*ref:C2
=1	=1	=1	=1	=1
4.231	.	.	.	27/8
6.182	2	.	.	.
7.441	.	.	.	3
=2	=2	=2	=2	=2
9.337	.	3	.	.
9.502	.	.	636	.
9.521	.	.	7/2	.
11.522	.	.	.	6
=3	=3	=3	=3	=3
13.571	.	.	7	.
13.958	.	.	.	9
14.814	.	.	.	6
15.358	.	.	12	.
15.457	1	.	.	.
15.567	.	.	.	27/2
=4	=4	=4	=4	=4
16.487	.	.	63/4	.
17.037	.	.	.	9
17.459	.	.	63/8	.
17.776	.	.	.	27/2
18.162	.	.	27/2	.
18.476	.	.	18	.
19.854	.	.	.	16
=5	=5	=5	=5	=5
20.038	.	2	.	.
20.223	.	.	21	.
20.594	.	.	.	12
20.837	.	.	27/2	.
22.064	.	.	.	18
22.103	.	.	27	.
22.11	.	.	9	.
22.816	.	.	.	27/4
23.334	.	.	14	.
23.673	.	.	.	12

Figure 9. First page of final performance score after quantization of the MIDI file compiled from by Ratioscore, along with original Ratioscore representation of the same music. Lines starting with an equal sign (=) represent barlines in the notation.

is instead controlled by pitch-bend messages.

A limitation of using pitch-bend to control tuning is that all notes on a particular channel share the same pitch-bend value.[8] With General MIDI, there are 16 channels with one reserved for percussion, so this means that 15 notes can have an independent tuning at any given time, with each monophonic voice assigned to its own channel.[10] Since the system was designed to prototype a string quartet, using MIDI files in this manner is not a problem and is compensated for by avoiding the need for specialized software or synthesizers.

3.1 Time quantization

To finalize rhythmic values of Ratioscore free timings, compiled MIDI files can be post-processed by other software at the discretion of composers who want to generate standard graphical notation. Most notation software and some digital audio workstations can rudimentarily quantize MIDI into metered, traditional Western notation rhythms. Some parameters can be controlled, such as the type of tuplets, minimum duration, and articulation (staccato or tenuto). OpenMusic, bach composer helper, and PWGL have plugins that can fine-tune quantization.

4. AS A COMPOSITIONAL TOOL

Ratioscore was in part developed out of necessity. We wanted to be able to hear the just-intonated composition written for the JACK quartet, *The Answers*.⁹ Here, the composition can serve as a compositional illustration of how Ratioscore might be used.

The Answers is a string quartet whose timings are derived from the rotation, orbit, and vantage point of the first four planets in our solar system. Sixteen points (ratios) were superimposed in a 4x4 square that circumscribes the orbit of Mars (cello) in Cartesian space.

The rhythm is determined by the rotation of each planet at their relative positions over the course of a year on Mars, scanning the points like a light beam sweeping across a square grid of stars, changing to each corresponding ratio. From each vantage point, the order and timing of sonified stars permute, which means the combined quartet of ratios in our pitch space morphs fluidly, though maintaining a constant insistence of the pitch space, centered around the fundamental C2.

These timings, exact to the thousandth second, and their corresponding ratios were calculated in Excel and exported as a textfile for Ratioscore. The Ratioscore converter created a MIDI file simulation of the composition, as well as for each part, which was then quantized in Sibelius. In Sibelius, the MIDI pitch bends were preserved, but they had to be removed when incorporating the HEJI accidentals because C-septimal flat appeared as an altered B \flat , and so on. When changing note names, the pitch bends disappear. Thus, the score in Sibelius is solely graphical. Figure 9 shows the first five measures of *The Answers* engraved using Sibelius from the MIDI files with articula-

tion adjustments and dynamic additions, as well as the Ratioscore equivalent text score for comparison.

5. CONCLUSIONS

Though the JACK quartet did not need the simulation, as they are experts in the just intonation field, the simulation was incredibly helpful in rehearsal preparation. And at the nascence of the piece, Ratioscore afforded easy experimentation with different tunings and timings. The rhythm of *The Answers* would be markedly different if it had to begin with meter rather than absolute seconds. If a ratio needed to be switched out, a simple search and replace quickly manifested the change, with immediate playback. For future work, Ratioscore would greatly benefit from an additional label to specify note names so that when the MIDI file is loaded into the notation software, pitch names could be preserved and thus Sibelius could playback the correct tunings even after the score is adjusted to accommodate HEJI accidentals.

6. REFERENCES

- [1] T. Nicholson and M. Sabat, “Fundamental Principles of Just Intonation and Microtonal Composition,” 2018.
- [2] R. A. Wannamaker, “The spectral music of James Tenney,” *Contemporary Music Review*, vol. 27, no. 1, pp. 91–130, 2008.
- [3] M. Sabat and W. Von Schweinitz, “The Extended Helmholtz-Ellis JI Pitch Notation,” *Plainsound Music Edition*, 2004.
- [4] T. Nicholson, “Plainsound Harmonic Space Calculator.” [Online]. Available: <https://www.plainsound.org/HEJI>
- [5] M. Sabat, “Microtonal notation in Dorico,” August 2018, [posted 28-August-2018]. [Online]. Available: <https://www.scoringnotes.com/reviews/microtonal-notation-in-dorico>
- [6] T. Nicholson, “Microtonal playback in Dorico,” September 2018, [by Mikhael Rimsky-Korsakov in comments section; posted 4-September-2018]. [Online]. Available: <https://www.scoringnotes.com/reviews/microtonal-playback-in-dorico>
- [7] D. Huron, et al., “Humdrum documentation.” [Online]. Available: <https://www.humdrum.org>
- [8] MIDI Association, “MIDI 1.0 Official Specification.” [Online]. Available: <https://www.midi.org/specifications/midi1-specifications>
- [9] —, “Standard MIDI Files.” [Online]. Available: <https://www.midi.org/specifications/file-format-specifications/standard-midi-files>
- [10] —, “General MIDI.” [Online]. Available: <https://www.midi.org/specifications/midi1-specifications/general-midi-specifications>

⁹ Documentation video: <https://youtu.be/Dx5l-9MUZ1o>