HARMONIC MAPS: INTERACTIVE VISUALIZATION OF TRIAD SPACES BASED ON SPECTRAL STRUCTURES

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ABSTRACT

We present Harmonic Maps, a visualization of three-note chord spaces and an interactive application that allows users to explore in real-time the connection between the visualization and its mapped sounds. While typical harmonic analysis is based only on notes or on an audio signal, our analysis takes a hybrid approach by quantifying different types of interactions between the spectra of notes. These quantifications, which we call Harmonic Descriptors, are derived from acoustic or perceptual models. Three such descriptors are defined and mapped: concordance, third order concordance and roughness.

Harmonic analysis based on spectral structures opens new possibilities beyond traditional note-only or signal-only approaches. They can be applied to a continuum of frequencies, independent of the tuning system, as well as historical and stylistic constraints. Harmonic Maps based on spectral structures can be especially relevant to study the relationship between timbre and harmony. Our interactive exploration of harmonic spaces can have applications for analytical, compositional and educational purposes.

1. INTRODUCTION

A chord is a simultaneous combination of notes, and 3-note chords (triads) are considered to be the basic building block in tonal harmony [1]. A common approach to chord analysis is functional harmony, which defines a role for each chord in a sequence (e.g. tonic, dominant), but its use is limited to chords within a musical context. To analyze chords independent of musical context, three main approaches have been taken, each considering the chord in a different way. One, which we call the combinatorial approach, considers chords as abstract objects and defines ways of classifying them using mathematical formalisms [2]. A contrasting approach, audio descriptors, considers a chord as a single audio signal, and uses signal processing techniques directly on the audio without considering the abstraction of notes [3]. Finally, considering a chord as a sensation in the listener, perceptive models have been created through studies with human subjects [4, 5, 6].

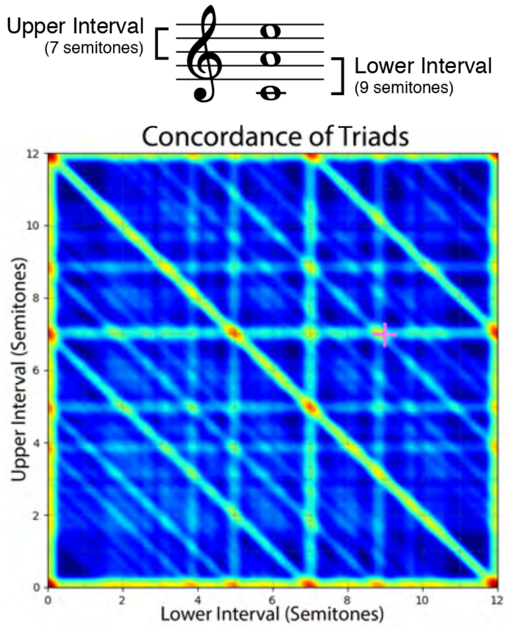


Figure 1. A triad and its equal-temperament position in a harmonic map for concordance for a synthesized sound with 11 partials. See Section 3.2 for the definition of concordance and section Section 4 for the implementation of Harmonic Maps.

Our work examines chords independent of harmonic context but differs from previous approaches in its hybrid nature that unites physical and symbolic approaches. We begin by computing the spectrum of each note, and analyze harmony based on the interactions between the spectra of different notes, with computational methods derived from acoustic and perceptual models. Our analyses are directly interpretable based on symbolic notions derived from the score, such as notes, intervals, and chords.

This article begins with a historic overview of interval, dyad and triad classifications (Section 2). We then present the notion of harmonic descriptors (Section 3), which are different ways of quantifying the spectral interactions between notes. Three harmonic descriptors are discussed, derived from acoustic and perceptual models: concordance, third order concordance, and roughness. Building on the notion of harmonic descriptors, we then focus on a specific type of visualization, the Harmonic Map, taking advantage of the fact that all possible three note chords based on a

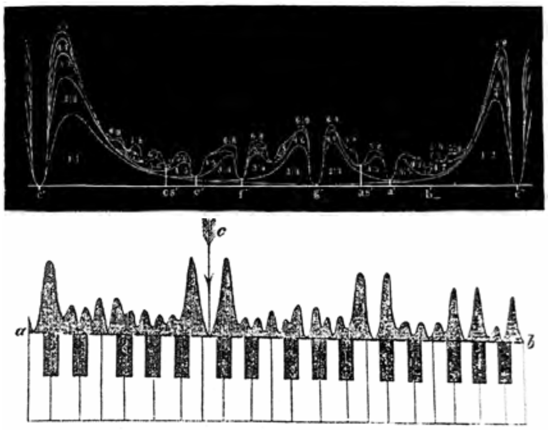


Figure 2. Description of intervals. (Top) Helmholtz's historical roughness curves, from [8]. (Bottom) Mach's vulgarisation of Helmholtz's curves, from [9].

chosen root note can be represented in a 2D plane, with the X and Y dimensions showing the lower and upper intervals. Assumptions and constraints of Harmonic Maps are discussed (Section 4), followed by the design and implementation of an interactive application enabling users to explore the connection between a visualizations and its sounds (Section 5). Finally, to illustrate the usefulness of our representation, Section 6 presents examples that shed light on topics in music theory.

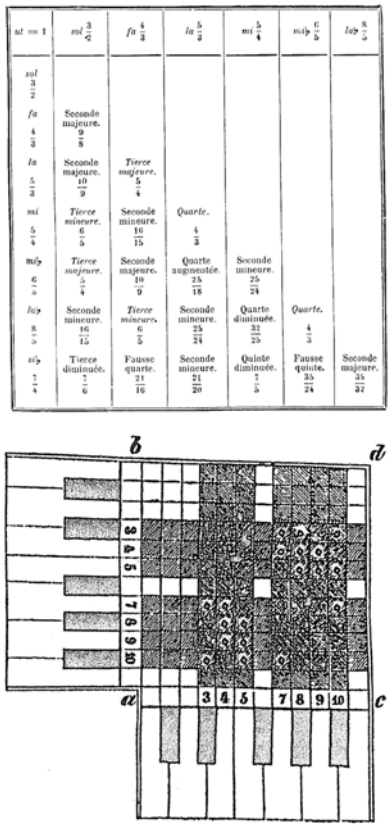
2. BACKGROUND

To situate the analysis of triads, we begin with the classification of intervals. An interval is defined as the set of dyads (two-note chords) whose two notes have the same logarithmic distance on the frequency axis. The classification of intervals is not new, and since the advent of polyphony in Western music, intervals have been classified into two categories: consonant and dissonant, whose meanings and criteria have evolved over history [7].

Historically, intervals were categorized based on discrete frequencies [7]. It is only with Helmholtz that an organization of intervals as a continuum was envisaged, via a psychoacoustic characterization, that links acoustic properties of sounds with their perception [8]. For example, the principle of "roughness" was developed through experiments where Helmholtz listened to different combinations of frequencies and counted the number of "beats" that arose in the frequency interactions. Figure 2 shows Helmholtz roughness curves, where the continuum of frequencies are shown on one axis, above which are the roughness values of the intervals for sounds containing different numbers of partials.

Three years after the publication of Helmholtz's seminal *Theory of Music*, Ernst Mach wrote a text popularizing Helmholtz's theory [9], with a new representation of Helmholt's roughness curve over two octaves, aligned with a piano keyboard with keys centered on their frequency (Figure 2). This representation shows the correspondance between the discrete and the continuous approach.

Despite his innovation in the description of intervals in continuous space, Helmholtz took a reductionist and categorical approach for the study of triads. The reductionist approach considers the interval as the basic building block of harmony, the study of which, both perceptual and theoretical, would suffice to understand higher cardinality chords [10]. Figure 3 shows a table of triads organized according to two constituent intervals: the lower interval and the one between the two extreme notes. The third interval is indicated in the table, and allows Helmholtz to determine a property of the triads, namely its consonance. Mach, in turn, takes the chord chart from Helmholtz by mapping it to the keys of the keyboard [9], with a code to indicate consonant intervals and chords (Figure 3).



ut = 1	sol 3/2	fa 4/3	la 5/3	mi 5/4	mi♭ 6/5	la♭ 8/5
sol 3/2						
fa 4/3	Seconde majeure. 9/8					
la 5/3	Seconde majeure. 10/9	*Tierce majeure.* 5/4				
mi 5/4	*Tierce mineure.* 6/5	Seconde mineure. 16/15	*Quarte.* 4/3			
mi♭ 6/5	*Tierce majeure.* 5/4	Seconde majeure. 10/9	Quarte augmentée. 25/18	Seconde mineure. 25/24		
la♭ 8/5	Seconde mineure. 16/15	*Tierce mineure.* 6/5	Seconde mineure. 25/24	Quarte diminuée. 32/25	*Quarte.* 4/3	
si♭ 7/4	Tierce diminuée. 7/6	Fausse quarte. 21/16	Seconde mineure. 21/20	Quinte diminuée. 7/5	Fausse quinte. 35/24	Seconde majeure. 35/32

Figure 3. Classification of triads (Top) Helmholtz's table of triads. (Bottom) Mach's vulgarisation of Helmholtz's curves, from [9].

From Mach's representation to a continuous representation of triads, there is only one step, which consists in reducing the size of note subdivisions by taking the limit. This step was only taken at the end of the 20th century, in the works of Chouvel and Sethares (Figure 4) [11, 12]. Chouvel takes the upper and lower intervals as axes, and uses level lines to represent concordance, a physical quantity that measures the energy of interaction. Sethares, meanwhile, uses the same axes as Helmholtz, with a 3D representation of roughness. Both Chouvel and Sethares base their work on theorical spectra and do not use real sounds.

The description of triads based on their constitutive intervals remains the majority approach in the psychoacoustic literature since Helmholtz, in spite of some proposals, to use the expression of Cook, that advocate a "psy-

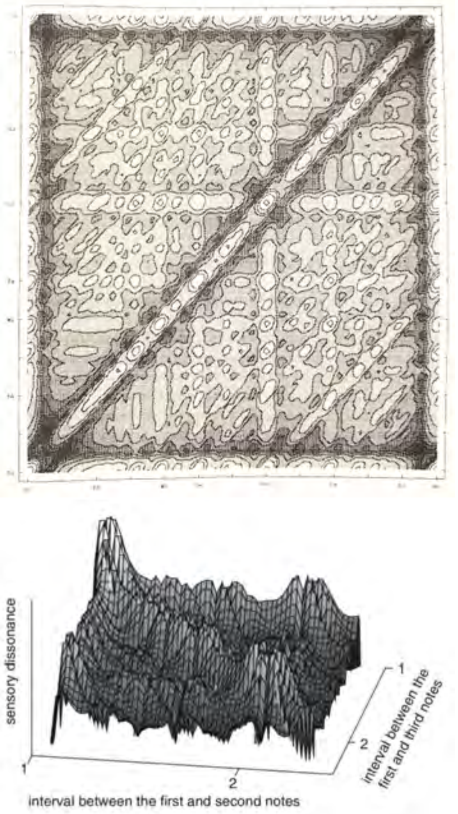


Figure 4. Continuous description of triads (Top) Chouvel, concordance map, from [11]. (Bottom) Sethares, 3D representation of sensory dissonance, from [12].

chophysics with 3 notes" [10]. Several models, however, exhibit acoustic and psychoacoustic properties unique to triads. This is the case of harmonicity [13], which measures the degree of similarity of a spectrum to a harmonic spectrum, of Cook's tension [14], which takes into account the differences between the upper and lower intervals, and of Gaulhiac's third order concordance [15], which quantifies the energy shared in the frequency ranges common to the three notes.

3. HARMONIC DESCRIPTORS

Harmonic descriptors are a set of notions and corresponding computation methods that aim to reconcile signal processing tools and perceptual models with a musicological understanding of music. The incorporation of signal processing tools that make calculations directly on acoustic reality allows us to take into account the timbre of different instruments, the articulation of playing, room acoustics, and to perform analysis on synthesized sounds. However, analysis based purely in signal-processing, such as audio descriptors can be difficult to interpret musically because they are often computed on a time-scale smaller than the duration of notes [3].

The novelty of harmonic descriptors is the association of each note with a spectrum, retaining a note-level description common in musicology while adding a signal-based layer. Each harmonic descriptor describes a type of interaction of a spectral nature between simultaneous notes. They are therefore ideal for the study of chord spaces, such as the triad space. The following sections first describe the general framework of how harmonic descriptors and how the spectra of a note is computed. We then present 3 specific descriptors: concordance, third order concordance and roughness.

3.1 Implementation & Spectra Computation

The implementation of our harmonic descriptors differs from the usual implementation of audio descriptors in that they are not calculated on the analysis windows but on the temporal duration of the chord. The values of the models are then associated with a chord, which can be more easily linked with a musical score. This paradigm shift makes the results readable and usable by musicologists.

Various ways of computing harmonic descriptors have been proposed, depending on the purpose of the analysis [15, 16]. For the purpose of computing our Harmonic Map of triads, each harmonic descriptor takes in three spectra, each associated with a note. A spectrum is computed from an initial reference sound, the sound of a note representative of the timbre that we wish to study. For the moment, the three spectra are generated from the same reference sound. This sound is projected in a time-frequency space by a constant-Q transform (CQT) [17]. This transform has the advantage, compared to the short-term Fourier transform, of preserving the same frequency precision on the logarithmic axis of the frequencies, to the detriment of the temporal precision in the bass register. Here we set a frequency precision of 1/32 of a tone for the analysis, i.e. each tone is divided into 32 equal frequency bands. The next step is to associate this sound with an "average spectrum" in amplitude, obtained by temporally averaging the amplitudes, frequency range by frequency range, over its entire duration. By carrying out such an averaging, the initial sound is approximated by a stationary sound, which puts aside the temporal monitoring of the spectral components' decrease. On the other hand, associating an amplitude spectrum with a note creates a bridge between the signal and symbolic notions, which makes it possible to calculate spectral models on chords. For the calculation on a triad, each note is associated with a spectrum, obtained by translation of the "average spectrum," and the harmonic descriptors are calculated on these spectra.

It is possible to imagine a simpler spectra computation method whereby the Fourier transform is taken on the initial sound without any windowing. However, our method was designed to ensure a constant precision in the frequency-logarithmic domain.

3.2 Concordance

The concordance between two stationary sounds is defined as the scalar product of their amplitude spectra. It quantifies the spectral components common to these two sounds. In the signal vector formalism, concordance is identified with the Hilbertian product in the frequency domain, in the particular case of real spectra. Bonnet interprets it as an energy of interaction between two sounds [18].

In all generality, for two complex spectra X_1 et X_2, there is the relation:

$$\|X^{(1)}+X^{(2)}\|^2 = \|X^{(1)}\|^2+\|X^{(2)}\|^2+2\Re\langle X^{(1)}, X^{(2)}\rangle, \quad (1)$$

where $\Re$ denotes the real part.

The term "harmonic concordance" was introduced by Chouvel, who uses it as a guide to composition, primarily in microtonal spaces [11]. The concordance $\mathcal{C}(X^{(1)}, X^{(2)})$ of two notes with amplitude spectra $X^{(1)}$ and $X^{(2)}$ is expressed as the scalar product of the spectra:

$$\mathcal{C}(X^{(1)}, X^{(2)}) = \langle X^{(1)}, X^{(2)}\rangle = \sum_k X_k^{(1)} X_k^{(2)}, \quad (2)$$

where $X_k^{(i)}$ is the k^{th} bin amplitude of $X^{(i)}$ and k goes through the frequency bins.

From there, the concordance $\mathcal{C}$ of a triad with notes' amplitude spectra $X^{(1)}$, $X^{(2)}$ and $X^{(3)}$ is defined as the sum of the concordances of all the pairs of notes:

$$\mathcal{C} = \mathcal{C}(X^{(1)}, X^{(2)}) + \mathcal{C}(X^{(1)}, X^{(3)}) + \mathcal{C}(X^{(2)}, X^{(3)}). \quad (3)$$

3.3 Third Order Concordance

The concordance of a triad measures the spectral components common to two notes of the triad. Therefore, it is natural to want to measure the spectral components common to the three notes simultaneously. This is the role of Gaulhiac's third-order concordance [15]. Third-order concordance is more restrictive than concordance, since it requires the simultaneous coincidence, and no longer two-by-two, of the partials resulting from the three notes. The term "third order" or "order 3" is added to refers to the simultaneous coincidences of the three notes, and to distinguish it from the notion of previously defined notion of concordance, which is of order 2.

The third order concordance of a triad with notes' amplitude spectra $X^{(1)}$, $X^{(2)}$ and $X^{(3)}$ is defined as follows:

$$\begin{aligned} \mathcal{C}_3 &= \langle X^{(1)}, X^{(2)}, X^{(3)}\rangle \quad (4) \\ &= \sum_k X_k^{(1)} X_k^{(2)} X_k^{(3)}. \end{aligned}$$

3.4 Roughness

The roughness of a sound is the sensation due to the beats between partials that it comprises. When two sinusoidal sounds are played simultaneously, they interact to form frequency beats equal to the difference in the frequencies of the initial sounds. When the frequencies are close enough, the beats are clearly audible, but when they move a little apart, they become too fast to remain audible but still create a feeling of harshness, called roughness. Moving further away, the frequencies are perceived as distinct, and the feeling of roughness disappears.

Research in psychoacoustics has shown that roughness is one of the causes in the judgment of dissonance. If Sauveur already explained dissonance by beats, Helmholtz, to whom we owe the term "roughness," is the first to experimentally study the link between the two, with a psychoacoustic approach that revolutionizes the theory of consonance and dissonance [8]. A century later, based on perceptual experiments, Plomp and Levelt relate roughness to the notion of the critical band, which gives an order of magnitude of the frequency difference below which two sinusoidal sounds are close enough to interact [4]. They propose a dissonance curve for simple sounds where the maximum dissonance corresponds to a frequency difference equal to a quarter of the critical band.

The results of Plomp and Levelt mark the beginning of a long series of models refining that of Helmholtz. We use a model largely inspired by that of Sethares [12], which is based on the curve of Plomp and Levelt.

The roughness $\mathcal{R}(\alpha_1, \alpha_2, f_1, f_2)$ of two sinusoidal sounds of frequency f_1 and f_2 and amplitude α_1 and α_2, with $f_1 \leq f_2$, is expressed as :

$$\begin{aligned} \mathcal{R}(\alpha_1, \alpha_2, f_1, f_2) = \alpha_1\alpha_2 \Big(&e^{-b_1(f_1-f_2)s(f_1)} \quad (5) \\ &-e^{-b_2(f_1-f_2)s(f_1)}\Big), \end{aligned}$$

with

$$s(f_1) = \frac{x^*}{s_1 f_1 + s_2}. \quad (6)$$

The constants b_1 and b_2 control the shape of the roughness curve (location of the roughness maximum and decay after the maximum), while s_1 and s_2 are linked with the variation of the critical band as a function of the lower frequency. We fix $b_1 = 3.5$, $b_2 = 5.75$, $s_1 = 0.021$ and $s_2 = 19$. The location of the maximum roughness in proportion to the critical band is indicated by $x^* = 0.24$.

From this, the roughness $\mathcal{R}$ of a triad with notes' amplitude spectra $X^{(1)}$, $X^{(2)}$ and $X^{(3)}$ is obtained by summing the roughness of all combinations of pairs of bins from different notes :

$$\mathcal{R} = \sum_{i<j}\sum_{k,l} \mathcal{R}(X_k^{(i)}, X_l^{(j)}, f_k, f_l), \quad (7)$$

where $X_k^{(i)}$ is the k^{th} bin amplitude of $X^{(i)}$, f_k the central frequency of bin k, and k and l go independently along the frequency axis.

4. FROM HARMONIC DESCRIPTORS TO HARMONIC MAPS

Harmonic Maps are a way of visualizing the values of a harmonic descriptor for triads, providing an acoustic or psychoacoustic description of the chord space depending on the descriptor used. A Harmonic Map is derived from a single reference sound, from which the values of a chosen descriptor are calculated for all chords with the reference note as the root (lowest) note. Input sounds with little noise and minimal reverberation are preferred. Values of the descriptor can be represented either with the addition of a third dimension or through colors (Figure 1 and 5). While harmonic descriptors can be calculated between intervals of any size, we limit the lower and upper intervals to one octave in the examples we present. The values of our descriptors are normalized between 0 and 1. Our implementation is based on several assumptions that pose constraints on the type of sounds used, to ensure that the Harmonic Maps we generate reflect an acoustic reality.

4.1 Stability of Sounds

The temporal average of the spectrum is representative of the acoustic reality of the basic sound only if the latter has a certain temporal stability. The model instrument is the organ, with its stable and sustained sounds, which offers a wide variety of timbres through its stops.

Temporal instability can arise from the decrease in the partials for sounds that are not maintained, from the attack phases, or from playing techniques such as tremolo or vibrato. When using sounds without temporal stability, the instability of the sound must be taken into account in the interpretation of the Harmonic Maps.

For example, to include the vibrato effect, one must ensure that the starting sound is long enough to contain enough periods of the vibrato. For sounds with a prominent attack, as for the piano, one can eliminate the transients of the attack by processing the type separation of the harmonic part and the percussive part upstream [19]. For a sound that is not sustained, such as the bell (Figure 7), the generated spectra will not reflect the decay in the sound, and the resulting map may be less reflective of acoustic reality. To evaluate the degree of relevance of a Harmonic Map, one can re-synthesize the averaged sound from the generated averaged spectrum. By comparing the averaged sound with the original sound, it is possible to judge by ear its degree of artificiality.

4.2 Timbral Considerations

The implementation of a Harmonic Map takes only one sound as input, and from it generates the spectrum which serves as a model for all notes. However, on acoustic instruments, the shape of the spectrum varies with pitch, and the visualization may be less accurate for notes further away from the reference note. This is another reason we limit our maps to the range of one octave.

Each Harmonic Map is currently limited to a single timbre, where a timbre is associated with a mean amplitude spectrum. A single Harmonic Map cannot take into account chords whose notes are played by different instruments, as can occur in an instrumental ensemble.

5. INTERACTIVE HARMONIC MAPS

We developed an application that enables users explore the connection between a Harmonic Map and the sounds that it describes. Our application supports several interfaces, including the mouse, the stylus of a graphic tablet, and MIDI or MPE (Midi Polyphony Expression) controllers. With a mouse or stylus, a user can hear what sounds correspond to different regions of the map by clicking on points or drawing a continuous trajectory (See figure 6). With a MIDI/MPE controller such as a keyboard, a user can play a chord or sequence of chords to see their corresponding location on the Harmonic Map.

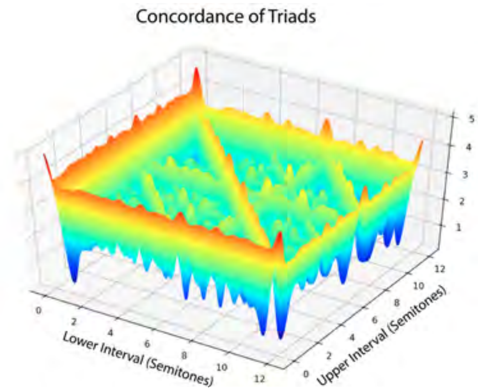


Figure 5. Harmonic Maps can also be visualized in 3D. Here is the 3D version of the concordance map shown in Figure 1, where both the color and height are determined by the concordance. 3D Harmonic Maps can also be used to visualize two descriptors at once.

Designed as an interactive tool for exploring a harmonic space, our application can be particularly useful for the composer or the musicologist who works with harmonic material for which there is not yet well-established theories, such as microtonal scales or inharmonic sounds. In addition, the comparison of the different structures associated with different timbres makes it possible to study and explain the timbre-harmony relationship, taking into account, for example, the influence of the instrument, the nuances, the modes of play.

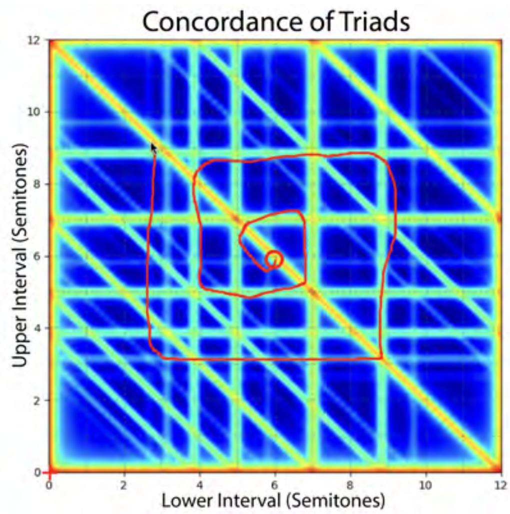


Figure 6. A trajectory on a Harmonic Map

5.1 Implementation

The Harmonic Maps application consists of two parts. First, a Python script manages the calculation of the descriptors and generates images of Harmonic Maps from an input sound. A Max patch then manages the real-time interaction, displaying a selected map and synthesizing sounds based on information from input devices. For the moment, the calculation of the descriptors and the generation of Harmonic Maps is not done in real time. The Python script for a pre-determined set of input sounds is run prior to running the Max patch, which then allows the user to select from a list of pre-computed maps.

For sound synthesis, we use the collection of sampled instruments from the Ircam Solo Instruments 2 library [1] , due to the many playing techniques available and its support of the MPE (MIDI Polyphonic Expression) protocol. For each timbre considered, a sound-model of a few seconds of a note is extracted, then injected into the Python script to build the associated Harmonic Maps.

5.2 MPE Control & Harmonic Trajectories

While traditional MIDI keyboards can be used with the Harmonic Maps application, they do not allow access to all regions of a map because the classic pitch-bend wheel transposes the entire keyboard. In order to control the harmonic trajectories by a MIDI controller, it is necessary to have an instrument supporting the MPE protocol. MPE keyboards such as the Roli Seaboard [2] or the Haken Continuum Fingerboard [3] enable each note to be modulated independent of other notes, allowing all points on a Harmonic Map to be visited. We tested the harmonic trajectory feature with a the Osmose piano from Expresse E [4] , in which each key can move laterally to modulate its pitch.

6. EXAMPLES

We describe a several examples of Harmonic Maps to show how they can inform the understanding of sound material. The first three examples (Sections 6.1-6.4) focus on concordance, and show the influence from the numbers of partials, timbre, playing mode, and degree of harmonicity. We then show a roughness map example (Section 6.5) and present a theoretical application of third order concordance concerning the minor chord (Section 6.6). All the maps presented below use middle C (261 Hz) as their input and show the space of triads with this note as the root.

6.1 Influence of the Number of Partials

The spectral structures of triad spaces depend on the spectral properties of the considered sounds, and first of all on the number of partials. Figure 10 shows concordance maps with harmonic sounds composed of 2, 3 and 5 partials respectively, as well as their associated spectra. A sound is said to be harmonic when these partials are multiples of the fundamental frequency. The sounds were generated by additive synthesis using the FAUST language [5] . A decrease of $1/\sqrt{k}$ in the amplitude of the partials was applied, where k is thte order of the partial.

The color code is as follows: cold colors correspond to low values of concordance, warm colors to high values, with red representing the maximum concordance values. We are interested in the positions of the local maxima, formed by segments and the intersections between these segments. Each segment corresponds to a coincidence between partials from different notes. The structure becomes more complex with more partials, since the number of interactions increases. With two partials separated by an octave, the triads highlighted are those containing unison or octave intervals, located on the periphery of the map, as well as those whose interval between the bass and the highest note high is the octave. Adding the third partial, an octave and a fifth above the root, emphasizes chords with a fifth or an octave plus a fifth. The addition of partial number 5, located two octaves and a major third above the fundamental, reveals the major chords (fundamental and tight position of coordinates 4-3) and minor (3-4). The highlighted chords are in just intonation, i.e. formed from the intervals naturally present in harmonic sounds. This is why the highlighted major chord has a major third slightly less than 4 semitones. Note the symmetry of the maps according to the large diagonal starting from the origin, a property of concordance, which does not distinguish major and minor chords in root position with closed voicing (within an octave).

6.2 Influence of Timbre

Figures 11 show the concordance maps of three different organ stops: the vox humana, the unda maris, and the tutti. The sounds come from UVI's Orchestral Suite library [6] . Differences in timbre are reflected in the harmonic structures.

The sound of unda maris has few partials compared to the others, so its concordance map structure is simpler. The set of reed pipes tuned slightly higher than the others has the particularity of producing slight beats, which are reflected on the map by the greater width of the segments. The structure is more complex for the vox humana. That of the tutti is essentially distinguished by more contrast, the local maxima of concordance being more marked.

From the point of view of the organist or the composer, each stop calls for a specific writing, and we do not play the same types of chords with the vox humana as with the tutti. Harmonic maps are a tool to study and deepen the understanding of the relationship between timbre and harmony.

6.3 Influence of Dynamics & Playinng Techniques

Different playing techniques on an instrument has an effect on the timbre and therefore on the harmonic structures. Figure 12 shows the concordance maps of the cello with differences in dynamics and vibrato: first piano playing without vibrato, then piano with vibrato, finally forte with vibrato. The sounds come from the Ircam Solo Instruments 2 library.

As with the unda maris, the vibrato has the effect of widening the segments. It also widens the partials on the spectrum. The strong nuance has the effect of enriching the

[1] https://www.uvi.net/ircam-solo-instruments-2
[2] https://roli.com/
[3] https://www.hakenaudio.com/
[4] https://www.expressivee.com/
[5] https://faustdoc.grame.fr/
[6] https://www.uvi.net/en/orchestral/orchestral-suite.html

spectral content by increasing the number of partials, which results in an enrichment of the map.

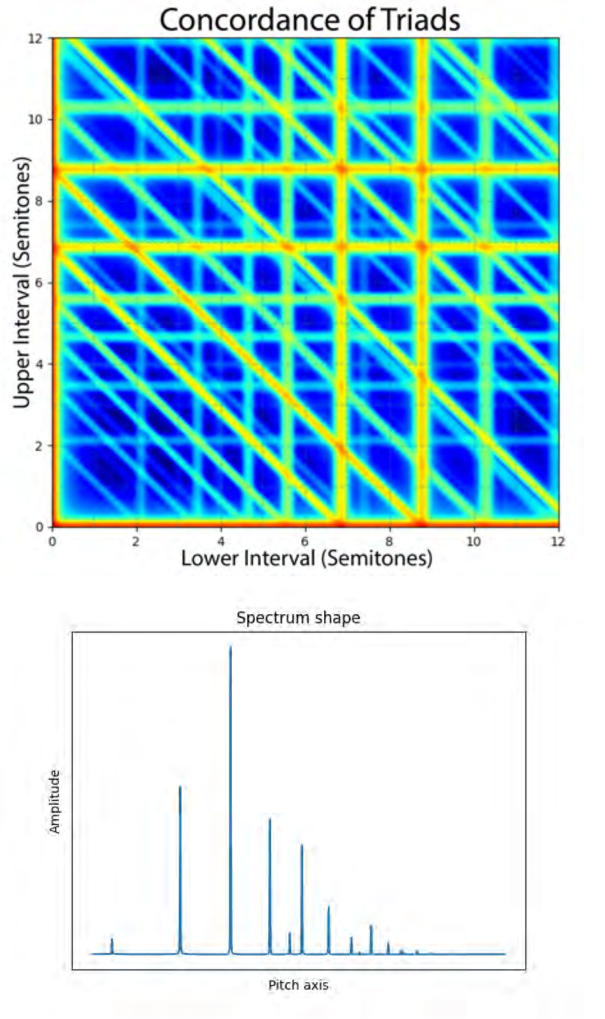


Figure 7. Concordance map for the tubular bell (Top), an inharmonic and non-sustained sound, and spectrum of this sound (Bottom).

6.4 Influence of Harmonicity

The spectra studied so far are harmonic spectra, but Harmonic Maps can also be constructed from inharmonic sounds, i.e. sounds whose partials are not multiples of the same fundamental frequency, such as from bells or the xylophone. Even in the case of very inharmonic sounds, for which it is complicated to perceive or define a fundamental frequency, the method remains applicable, since the spectra of the notes are obtained by the translation of the reference spectrum.

Figure 7 shows the concordance map of a tubular bell, an instrument used in the orchestra. The sound comes from UVI's Orchestral Suite library. The structure is very different from the previous structures. The absence of a segment marked on the edges of the map at the top and on the right shows that the instrument is not octaviant, in other words the spectrum does not include an octave interval on the first partials. The large over-diagonal shows that the octave equivalent for this sound is slightly larger than the 12 semitone interval. Likewise, the fifth is slightly lower than the interval of 7 semitones.

The sound exploration of this map by the Harmonic Maps application seems particularly relevant to guide the ear. The major chord on the first C-Eb-Ab inversion, with coordinates (3-5), is located on a blue part, that is to say a trough of concordance, and sounds very hard to the ears of authors, while the C-D-A chord, with coordinates (2-7), is close to a local maximum and has a softer sonority.

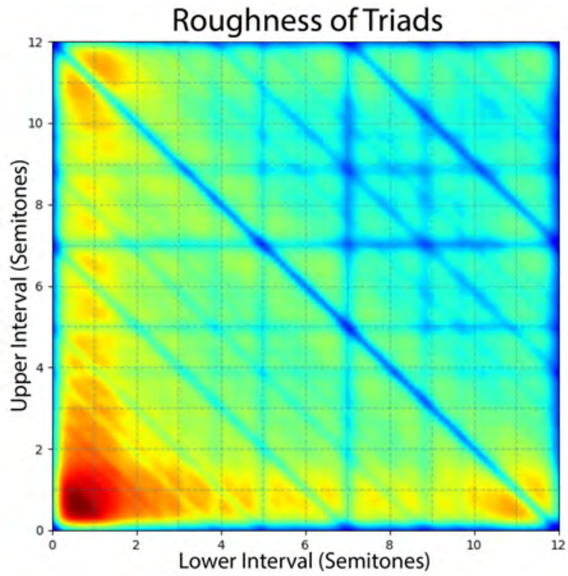


Figure 8. Roughness harmonic map for a synthesized sound with 11 partials.

6.5 Roughness

Roughness is a harmonic descriptor that reflect perceptive models, in contrast to concordance, a purely acoustic descriptor, which has not been the subject of perceptual studies.

Figure 8 shows the roughness map of a synthetic sound with 11 partials, with a very different structure from the concordance maps. The red zone on the triads formed by small intervals corresponds to a maximum of roughness. Roughness distinguishes major and minor chords in extended position C-G-E, with coordinates (7-9), and C-A-E, with coordinates (9-7), which correspond to local minima, but does not distinguish major and minor chords in tight voicings.

6.6 Third Order Concordance

Concordance and roughness are reductionist models, in the sense that they reduce the study of triads to the study of their constitutive intervals. The symmetry of concordance and roughness maps comes from this, since two symmetrical triads along the long diagonal have the same set of constituent intervals (the C-E-G and C-Eb-G triads are both made up of a fifth, a major third and a minor third). Third order concordance differs from concordance and roughness in that it is not reductionistic, as shown by the lack

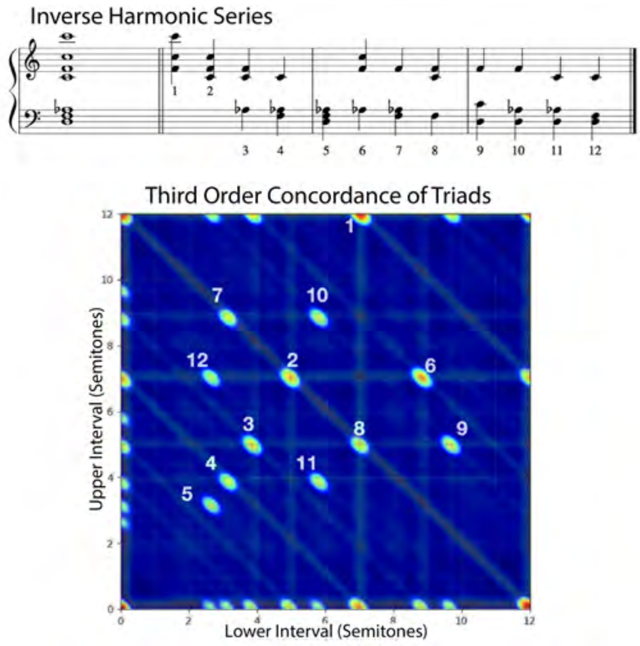


Figure 9. Example of a third order concordance map for a synthesized sound with 7 partials (Bottom) with a corresponding inverse harmonic series (Top). The numbered chords in the series are shown by their numbers in the map.

of such symmetry in the third order concordance map of a synthesized sound with 7 partials figure 9.

The local maxima of the third order concordance correspond to the triads for which there is a simultaneous coincidence of partials resulting from the three notes. The frequencies of the fundamental notes then divide this common frequency. In other wards, the triad is contained in the inverse harmonic series resulting from this common frequency, where the inverse harmonic series is defined as the series of frequencies which divide a given frequency. An inverse harmonic series, which does not have the physical reality of the harmonic series, is shown in figure 9, with the first 12 chords from this series. Reduced to the same lower note, they are projected on the map, and correspond exactly to the local maxima. Just as the harmonic series contains the major chords, the inverse harmonic series contains the minor chords, which are effectively identified by the third order concordance, while the minor chords are not.

7. CONCLUSIONS & FUTURE WORK

Harmonic Maps is a new way to visualize the spectral structure of triads in a continuous space, based on real sounds, which maintains the link with symbolic notions of notes and chords. Such a hybrid approach takes advantage of signal processing methods but makes their results more applicable for musicological purposes. Musicological subjects on which Harmonic Maps can shed light include the relationship between timbre and harmony, microtonality, and the study of non-harmonic sounds. We presented an interface for exploring the connection between Harmonic Map visualizations and the sounds they depict, allowing users to hear sounds by moving around on a map with a mouse or stylus, or by playing notes on a keyboard and seeing their trajectory on the map. Our visualization contributes to the literature of music theory and analysis, and the interactive interface makes a contribution for compositional and educational contexts.

In terms of future work, we plan to add a drag and drop feature, where new sound recordings can be uploaded to generate new maps without leaving the Max patch. A more ambitious future feature is the ability to generate maps based on real-time input sounds, which will involve optimizing our harmonic descriptor calculations to run in real-time. We can also imagine Harmonic Maps that can accomodate sounds played by different instruments.

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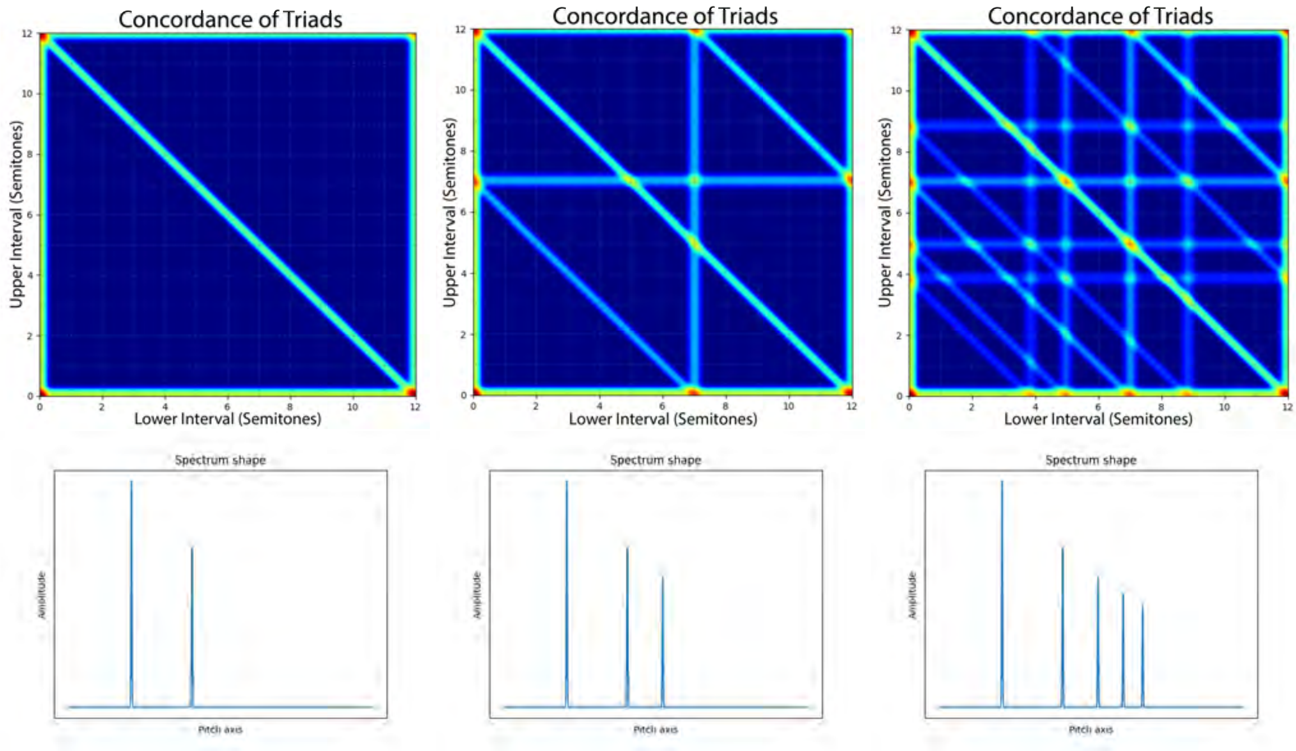

Figure 10. Influence of the number of partials. (Top) Concordance map for synthesized sounds with 2 (Left), 3 (Middle) and 5 (Right) partials. (Bottom) Corresponding spectra.

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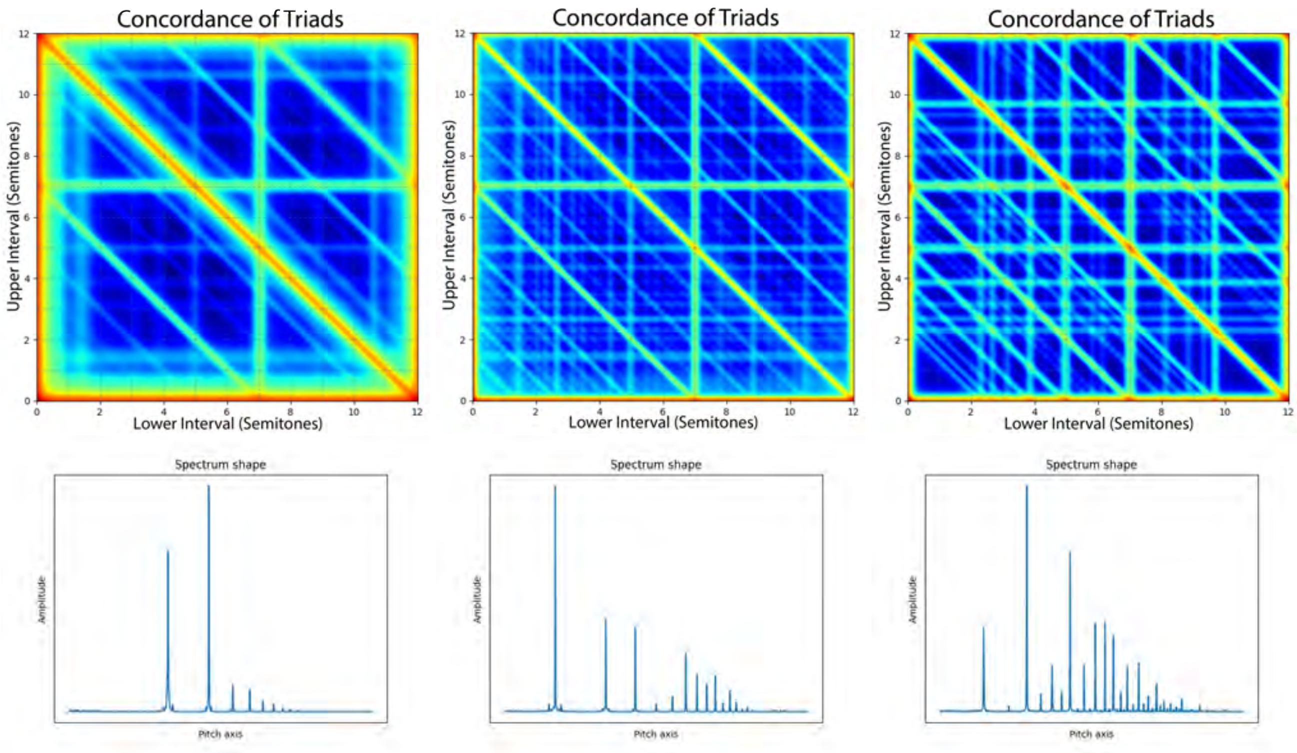

Figure 11. Influence of timbre. (Top) Concordance map for 3 different organ stops: unda maris (Left), vox humana (Middle) and tutti (Right). (Bottom) Corresponding spectra.

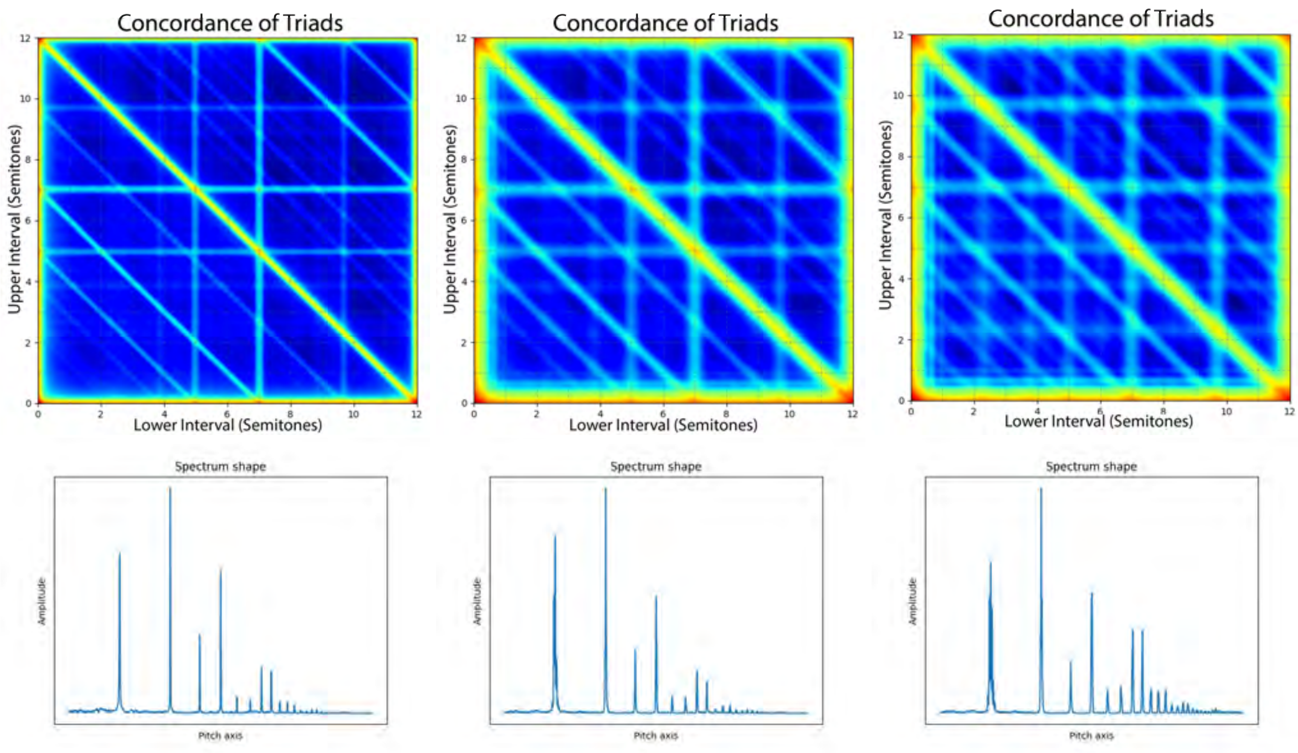

Figure 12. Influence of dynamics and playing technique. (Top) Concordance map for 3 different ways of playing on cello: nuance piano without any vibrato (Left), nuance piano with vibrato (Middle) and nuance forte with vibrato (Right). (Bottom) Corresponding spectra.